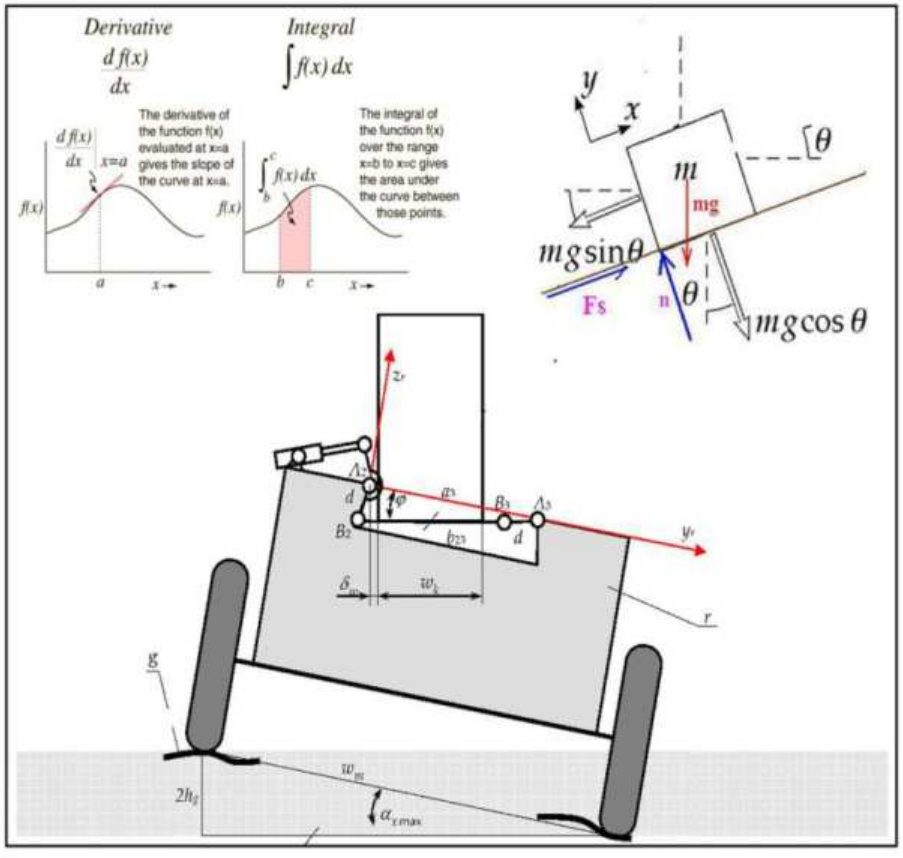
மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள்
நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 7 வது

முன்னோடிப் பரீட்சை - 2016

10(I) - இணைந்தகணிதம் I

விடைகள் (புள்ளியிடும் திட்டம்)



Prepared By
P.Senthilnathan *B.Sc. Dip in Edu*

MORA ENGINEERING FACULTY TAMIL STUDENTS | EXAMIANTION COMMITTEE

Part - A

01. $f(x) = 2.7^n + 3.5^n$ எனக் கொள்க.

$f(1) = 2\times 7 + 3\times 5$

$= 14 + 15$

$= (24 \text{ x}1) + 5$

∴ n=1 இற்கு முடிவு உண்மை (5)

$n = P(\in Z^+)$ இற்கு முடிவு உண்மை எனக் கொள்க.

$f(p) = 2.7^P + 3.5^P$

$= 24k + 5 \ (k \in Z)$ (5)

n = p +1 இற்கு

$f(P+1) = 2.7^{(P+1)} + 3.5^{(P+1)}$ (5)

$= 2.7.7^P + 3.5.5^P$

$= (12+12)7^P + 24k + 5$

$= 24m + 15 \ (m \in Z)$

∴ n = (p+1) இற்கு முடிவு உண்மை. (5)

∴ n = 1 முடிவு உண்மை, n = p முடிவு உண்மை எனில் n = (P + 1) முடிவு உண்மை ஆகவே f(x) இனை 24 இனால் வகுக்க மீதி 5 ஆகும். (5)

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02. $y = f(x) = x^2 - 4x + 3$

$y = f(x) = (x-2)^2 - 1$

திரும்பற்புள்ளி (2, -1) (5)

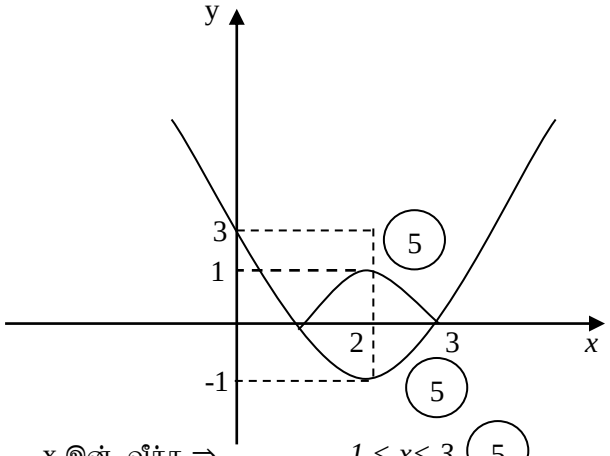
$y = 0 \Rightarrow \quad (x-2)^2 = 1$

$(x-2) = \pm 1$ (5)

$x = 3,1$

$x = 0 \Rightarrow \quad y = 3$

$x \to \pm\alpha$ எனின் $y \to \pm\alpha$



x இன் வீச்சு ⇒ $1 < x < 3$ (5)

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03. $Z = x + iy$ என்க

$|Z + 2 + 3i| = |1 - Z|$

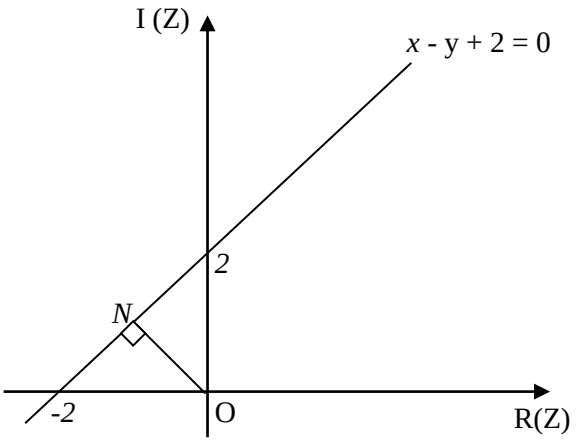
$|x + iy + 2 + 3i| = |1 - (x + iy)|$

$|(x+2) + (y+3)i| = |(1-x) - iy|$ (5)

$\Rightarrow \sqrt{(x+2)^2 + (y+3)^2} = \sqrt{(x-1)^2 + (-y)^2}$

$x^2 + 4x + 4 + y^2 + 6y + 9 = 1 - 2x + x^2 + y^2$

$6x - 6y + 12 = 0$



$x - y + 2 = 0$ (5)

$|Z|_{\min} = ON$

$= \dfrac{|0 - 0 + 2|}{\sqrt{1+1}}$

$= \dfrac{2}{\sqrt{2}}$

$= \sqrt{2}$ (5)

$No \Rightarrow x + y = 0$
$x - y + 2 = 0$ $\Rightarrow$ N(-1,1) (5)

$\therefore$ $|Z|$ இழிவாக இருக்கும். $Z = -1 + i$ (5)

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04. $\left(2x^3 - 3/x^2\right)^{15}$

$T_{r+1} = {}^{15}C_r (2x^3)^{15-r} \left(-3/x^2\right)^r$

$= {}^{15}C_r \, 2x^{15-r} x^{[3(15-r)-2r)} (-3)^r$

x ஐச் சாராத உறுப்பு பெறுவதற்கு

$3(15-r) - 2r = 0$

$45 - 5r = 0$

$r = 9$

$\therefore$ x ஐச் சாராத உறுப்பு

$r = 9 \Rightarrow T_{10} = {}^{15}C_9 (2x^3)^{15-9} \left(-3/x^2\right)^9$

$T_{10} = 15C_9 . 2^6 . (-3)^9$

$\left(2x^3 - 3/x^2\right)^{15} = {}^{15}C_0 \left(2x^3\right)^{15} \left(-3/x^2\right)^0 + {}^{15}C_1 \left(2x^3\right)^{14} \left(-3/x^2\right) + {}^{15}C_2 \left(2x^3\right)^{13} \left(-3/x^2\right)^2 + ...$

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05. $\lim\limits_{x \to 0} \dfrac{1 - \cos^2(3\sin x)}{(1 - \cos 2x)}$

$\lim\limits_{x \to 0} \dfrac{Sin^2(3\sin x)}{2Sin^2 x}$ (5)

(5) $\lim\limits_{\sin x \to 0} \dfrac{9}{2} \left(\dfrac{Sin^2(3\sin x)}{(3Sinx)^2} \right)$ (10)

$\dfrac{9}{2}(1)^2 = \dfrac{9}{2}$ (5)

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06. $y^2 = x(2-x)^2$

$y^2 = x(x2 - 4x + 4)$

$y^2 = x^3 - 4x^2 + 4x$ (1)

$2y\,dy/dx = 3x^2 - 8x + 4$ (5)

தொடலியின் படித்தின் $= \left(dy/dx\right)_{(1,1)}$

$= \dfrac{3-8+4}{2\times 1}$

$= (-1/2)$ (5)

தொடலியின் சமன்பாடு $\Rightarrow y-1 = -1/2\,(x-1)$

$2y-2 = -x+1$

$x+2y-3=0$ (5)

$(1),(2) \Rightarrow \quad \dfrac{(3-x)^2}{4} = x^3 - 4x^2 + 4x$

$x^2 - 6x + 4 = 4x^3 - 16x^2 + 16x$

$4x^3 - 17x^2 + 22x - 9 = 0$ (5)

$(x-1)(4x^2-13x+9) = 0$

$(x-1)(4x-9)(x-1)=0$

$X = 1 , 9/4$

$Y=1 , 3/8$

$P = (9/4 , 3/8)$ (5)

07. $AC \Rightarrow x - y + 10 + \lambda(x+3y-14) = 0$ (5)

$C(\bar{x},\bar{y}) \Rightarrow \bar{x} - \bar{y} + 10 + \lambda(\bar{x}+3\bar{y}-14) = 0$

$-2+10+\lambda(2-14) = 0$

$8 = 12\lambda \Rightarrow \lambda = 2/3$

$\therefore AC \Rightarrow x - y + 10 + 2/3\,(x+3y-14) = 0$ (5)

$5x + 3y + 2 = 0$

$BD \Rightarrow x - y + 10 + \mu(x+3y-2) = 0)$ (5)

$D(x_0,y_0) \Rightarrow x_0 - y_0 + 10 + \mu(x_0+3y_0-14) = 0)$

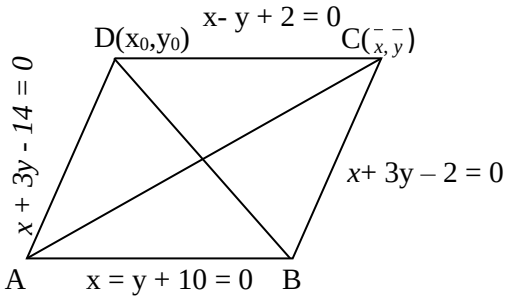
$-2+10+\mu(14-2) = 0$

$\mu = -8/12 = -2/3$

$8 = 12\lambda \Rightarrow \lambda = 2/3$ } (5)

$BD \Rightarrow x - y + 10 - 2/3\,(x+3y-2) = 0$

$\Rightarrow x - 10y + 34 = 0$ (5)



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08. $S \equiv x^2 + y^2 - 4x + 6y - 3 = 0$

$(x-2)^2+(y+3)^2-16=0$

$(x-2)^2+(y+3)^2=4^2$

மையம் $\equiv(2,-3)$ (5)

ஆரை $=4$ (5)

பொது தொடலியின் சமன்பாடு $\Rightarrow S_1-S$

$6x-8y+\lambda+3=0$ (5)

ஆரை $4=\dfrac{|6\times2-8\times(-3)+\lambda+3|}{\sqrt{(6)^2+(8)^2}}$ (5)

$40=|39+\lambda|$

$\pm40=|39+\lambda|$ $\qquad\qquad$ $\lambda=1,(-79)$ (5)

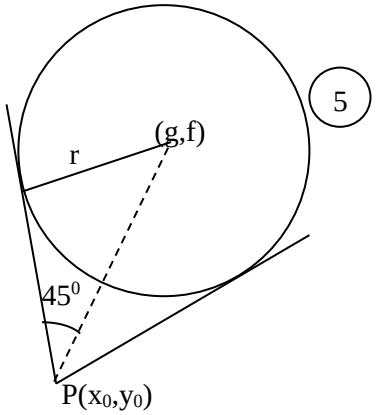
09. $Sin45=\dfrac{r}{PC}$ (5)

$PC=\sqrt{2}r$ (5)

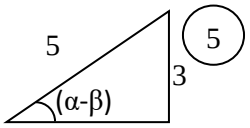
$\sqrt{(x_0+g)^2+(y_0+f)^2}=\sqrt{2}\sqrt{g^2+f^2-c}$ (5)

$(x_0+g)^2+(y_0+f)^2=2(g^2+f^2-c)$

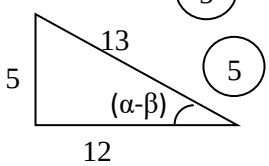
$\Rightarrow g^2+f^2=x_0^2+y_0^2+2gx_0+2fy_0+2c$ (5)



10. $0<(\alpha+\beta)<\pi/2$ $\qquad$ $0<\alpha+\beta<\pi/2$ (5)



(5)



(5)

$2\alpha=(\alpha+\beta)+(\alpha-\beta)$

$\tan(2\alpha)=\tan(\alpha+\beta)+(\alpha-\beta)$

$=\dfrac{\tan(\alpha+\beta)+(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$ (5)

$=\dfrac{3/4+5/12}{1-3/4\times5/12}$

$=\left(\dfrac{56}{33}\right)$ (5)

Part - B

11. (a) $F(x) \equiv (3x^2 - \alpha x + 3)(3x^2 - \beta x + 3)$

$G(x) \equiv x^4 - 2x^3 + 4x^2 - 3x + \lambda$

$H(x) \equiv x^4 + x^2 + 1$

i. $F(x) = 0$

$(3x^2 - \alpha x + 3)(3x^2 - \beta x + 3) = 0$

$9x^4 - 3(\alpha + \beta x)x^3 + (18 + \alpha\beta)x^2$

$-3(\alpha + \beta)x + 9 = 0$ (10) …(1)

$G(x) = 0$

$\Rightarrow x^4 - 3x^3 + 4x^2 - 3x + \lambda = 0$ (5) ……(2)

(1),(2) என்பன ஒரே மூலகங்களைக் கொண்டிருப்பதால்.

$\frac{9}{1} = \frac{-3(\alpha+\beta)}{-3} = \frac{18+\alpha\beta}{4} = \frac{-3(\alpha+\beta)}{-3} = \frac{9}{\lambda}$ (5)

$9 = (\alpha + \beta) = \frac{18 + \alpha\beta}{4} = \frac{9}{\lambda}$ (5) …….(3)

$9 = \frac{9}{\lambda} \Rightarrow \lambda = 1$ (5)

$D \Rightarrow \alpha + \beta = 9$ (5)

$(1) \Rightarrow 9 = \frac{18 + \alpha\beta}{4}$

$\alpha\beta = 18$ (5)

$\therefore$ (α, β) ஐ மூலகங்களாக உடைய சமன்பாடு $(x - \alpha)(x - \beta) = 0$

$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$ (5)

$x^2 - 9x + 18 = 0$

$(x - 6)(x - 3) = 0$

$x = (3,6) = (\alpha, \beta)$

G(x) =0 இன் மூலங்கள் F(x) =0 இன் மூலங்களாகும்.

$\Rightarrow (3x^2 - \alpha x + 3)(3x^2 - \beta x + 3) = 0$

$3x^2 - \alpha x + 3 = 0$ Or $3x^2 - \beta x + 3 = 0$

$\Rightarrow 3x^2 - 3x + 3 = 0$ Or $3x^2 - 6x + 3 = 0$

$x^2 - x + 1 = 0$ (5) $x^2 - 2x + 1 = 0$

$\Delta = 1 - 4 < 0$ (5) $(x - 1)cx - = 0$

$\therefore$ கற்பனை மூலங்கள் x = 1,1 (5)

$\therefore$ G(x) =0 இன் மூலங்களின் இரண்டு சமனான மெய் மூலங்கள் மற்றயஇரண்டும் கற்பனையானவை.

ii. $F(x) \equiv 9H(x)$ ……..(1)

$9x^4 - 3(\alpha+\beta)x^3 + (18+\alpha\beta)x$

$+9 \equiv 9(x^4 + x^2 + 1)$

$3(\alpha+\beta)x^3 + (18+\alpha\beta)x^2 - 3(\alpha+\beta)x \equiv 9x^2$ (5)

$[x^3] \Rightarrow 3(\alpha+\beta) = 0$

$\alpha + \beta = 0$ ……..(4) (5)

$[x^2] \Rightarrow 18 + \alpha\beta = 9$

$\alpha\beta = -9$

$\alpha(-\alpha) = -9$ From (4)

$(4) \Rightarrow \beta = -\alpha = \mp 3$ (5)

$\therefore \alpha = \pm 3, \beta = \mp 3$ (5)

H(x) =0

$\Rightarrow F(x) = 0;$ $From(I)$

$\Rightarrow (3x^2 - \alpha x + 3)(3x^2 - \beta x + 3) = 0$

$(3x^2 - x + 3)(3x^2 + 3x + 3) = 0$

$(x^2 - x + 1)$ $Or(x^2 + x + 1) = 0$

$\Delta = (1)^2 - 4\times1\times1$ $\Delta = 1 - 4\times1\times1$

$= -3 < 0$ (5) $\Delta < 0$ (5)

கற்பனை கற்பனை

∴ H(x) =0 இன் மூலங்களின் மெய்யானவை அல்ல

(b) $f(1)$ =0

a + b + c + 1 – 10 =0

a + b + c = 9 ……..(1) (5)

$f(2)$ = 0

16a + 8b + 4c + 2 – 10 =0

4a + 2b + c = 2 ……..(2) (5)

$f(-1)$ = 48

a – b + -1 -10 = 48 (5)

a + c – b = 59 ……(3) (5)

(1),(3) ⇒ a + c = 34 ……(*)

(1),(2) ⇒ c – 2a = 16 ……(**)

3a = 18

a = b

c = 28

b = (-25) (15)

$f(x) \equiv 6x^4\Delta - 25x^3 + 28x^2 + x - 10$

$(2x+1) \Rightarrow f(-½) = 6\times(-½)^4 - 25(-½)^3 + 28\times(-½)^2 + (-½) - 10$ (5)

$= \frac{3}{8} + \frac{25}{8} - 3 - ½$

$= \frac{3+25-28}{8}$

$= 0$ (5)

$f(x) = 6x^4 - 25x^3 + 28x^2 + x - 10$

$f(x) = (x-1)(x-2)(2x+1)(Ax+B)$

A = 3

B = (-5)

$f(x) = (x-1)(x-2)(2x+1)(3x+5)$

$= (x-1)(4x^2 - 13x + 9) = 0$

$= (x-1)(4x-9)(x-1) = 0$ (5)

12. (a)

விஞ்ஞானம்		கலை		வர்த்தகம்		தொழிற்நுட்பம்	
B	G	B	G	B	G	B	G
3	2	6	2	4	3	2	1

i. 1 தலைவர் + 1க(B) = 1க (G) + 2 வேறு $= 5_{C_1} \times 6_{C_1} \times 2_{C_1} \times 10_{C2}$ (10)

= 5 x 5 x 2 10 = 2700

1 தலைவர் + 1 (B) = 2 க (G) + 1 வேறு $= 5_{C_1} \times 6_{C_1} \times 2_{C2} \times 10_{C_1}$ =300 (10)

1 தலைவர் + 2 க(B) = 2 க (G) $= 5_{C_1} \times 6_{C_1} \times 2_{C1} \times 10_{C_1}$ =1500 (10)

1 தலைவர் + 2 க(B) = 1 க (G) $= 5_{C_1} \times 6_{C2} \times 2_{C2}$ =75 (5)

1 தலைவர் + 3 க(B) = 1 க (G) $= 5_{C_1} \times 6_{C3} \times 3_{C1}$ =300 (10)

மொத்தம் = 4875 (5)

ii. 1 தலைவர் + 1 கலை + 1 வர்த்தகம் + 2 தொ.நு

$= 5_{C_1} \times 8_{C_1} \times 7_{C1} \times (2_{C_1} \times 1_{C_1}) =$ 560 (5)

1 தலைவர் + 1 கலை + 2 வர்த்தகம் + 1 தொ.நு

$= 5_{C_1} \times 8_{C_1} \times (4_{C1} \times 3_{C_1}) \times 3_{C_1} =$ 1440 (5)

1 தலைவர் + 2 கலை + 1 வர்த்தகம் + 1 தொ.நு $= 5_{C_1} \times (6_{C_1} \times 2_{C_1}) \times 7_{C1} \times 3_{C_1}$

=180 (5)

மொத்தம் =3180 (5)

(b) $f(r) = r^2$ என்க.

$f(r) - f(r-1) = r^2 - (r-1)^2$

$f(r) - f(r-1) = 2r - 1$ (5)

$r = 1 \Rightarrow f(1) - f(2) = 2 \times 1 - 1$

$r = 2 \Rightarrow f(2) - f(1) = 2 \times 2 - 1$

$r = 3 \Rightarrow f(3) - f(2) = 2 \times 3 - 1$ (5)

$r = n-1 \Rightarrow f(n-1) - f(n-2) = 2(n-1) - 1$

$r = n \Rightarrow f(n) - f(n-1) = 2n - 1$ (5)

$$f(n) - f(0) = 2\sum_{r=1}^{n} r - n$$

$$n^2 - 0 = 2\sum_{r=1}^{n} r - n$$

$$n^2 - n = 2\sum_{r=1}^{n} r$$

$$\frac{n(n+1)}{2} = \sum_{r=1}^{n} r \quad (5)$$

$$\left[\frac{r(r+1)}{2}\right]^2 - \left[\frac{(r-1)r}{2}\right]^2 = \frac{r^2(r+1)^2}{4} - \frac{(r-1)^2 r^2}{4} \quad (5)$$

$$= \frac{r^2(r^2+2r+1) - r^2(r^2-2r+1)}{4}$$

$$= \frac{2r^3 - (-2r^3)}{4}$$

$$= r^3 \quad (5)$$

$let f(r) = \left[\frac{r(r+1)}{2}\right]^2$

$f(r) - f(r-1) = r^3$

$r = 1 \Rightarrow f(1) - f(0) = 1^3$

$r = 2 \Rightarrow f(1) - f(1) = 2^3$ (5)

$r = 3 \Rightarrow f(3) - f(2) = 3^3$

$r = n-1 \Rightarrow f(n-1) - f(n-2) = (n-1)^3$

$r = n \Rightarrow f(n) - f(n-1) = n^3$ (5)

$$f(n) - f(0) = \sum_{r=1}^{n} r^3$$

$$\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2 \quad (5)$$

$let\ Ur = r^3 - 3r + 1$

$\therefore U_{r+1} = (r+1)^3 - 3(r+1) + 1$ (5)

$U_r + U_{r+1} = r^3 - 3r + 1 + (r+1)^3 - 3(r+1) + 1$ (5)

$$= r^3 - 3r + 1 + r^3 + 3r^2 + 3r + 1 - 3r - 3 + 1$$

$$= 2r^3 + 3r^2 - 3r$$

$U_r + U_{r+1} = r(2r^2 + 3r - 3)$ (5)

$\therefore U_r = r^3 - 3r + 1$

ii. $U_r = r^3 - 3r + 1$

$$\sum_{r=1}^{n} U_r = \sum_{r=1}^{n} \left(r^3 - 3r + 1\right) \quad (5)$$

$$= \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$

$$= \left(\frac{n(n+1)}{2}\right)^2 - \frac{3n(n+1)}{2} + n \quad (5)$$

$$= \frac{n^4 + 2n^3 + n^2 - 3n^2 - 3n + 2n}{2} \quad (5)$$

$$= \frac{n^4 + 2n^3 - 2n^2 - n}{2} \quad (5)$$

13. (a) $C = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$ (5)

$$C^2 = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & -6 & 12 & -3 \\ -8+2 & & -6+1 & \end{pmatrix} \quad (5)$$

$$C^2 = \begin{pmatrix} 10 & 9 \\ -6 & -5 \end{pmatrix}$$

i.
$$C^2 - 3C + 2I = \begin{pmatrix} 10 & 9 \\ -6 & -5 \end{pmatrix} - 3\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix} + 2\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 9 \\ -6 & -5 \end{pmatrix} + \begin{pmatrix} -12 & -9 \\ 6 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (5)$$

$C^2 - 3C + 2I = 0$

$2I = 3C - C^2$

$= C(3I - C)$

$I = \frac{1}{2}C(3I - C)$

$I = C\left(\frac{3}{2}I - \frac{1}{2}C\right)$ (5)

$\Rightarrow C^{-1}I = C^{-1}\left[C\left(\frac{3}{2}I - \frac{1}{2}C\right)\right]$

$C^{-1} = \left(C^{-1}C\right)\left(\frac{3}{2}I - \frac{1}{2}C\right)$ (5)

$= I\left(\frac{3}{2}I - \frac{1}{2}C\right)$

$= \frac{3}{2}I - \frac{1}{2}C$

$= \frac{3}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$ (5)

$$= \begin{pmatrix} \frac{3}{2} & 0 \\ 0 & \frac{3}{2} \end{pmatrix} + \begin{pmatrix} -2 & -\frac{3}{2} \\ 1 & \frac{1}{2} \end{pmatrix} \quad (5)$$

$$C^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ 1 & 2 \end{pmatrix} \quad (5)$$

ii.

$$AB = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2+2 & 2+3 \\ 0+6 & 0+3 \end{pmatrix} \quad (5)$$

$$AB = \begin{pmatrix} 0 & 5 \\ 6 & 3 \end{pmatrix} \quad (5)$$

$$CX - AB = 0$$

$$CX = AB$$

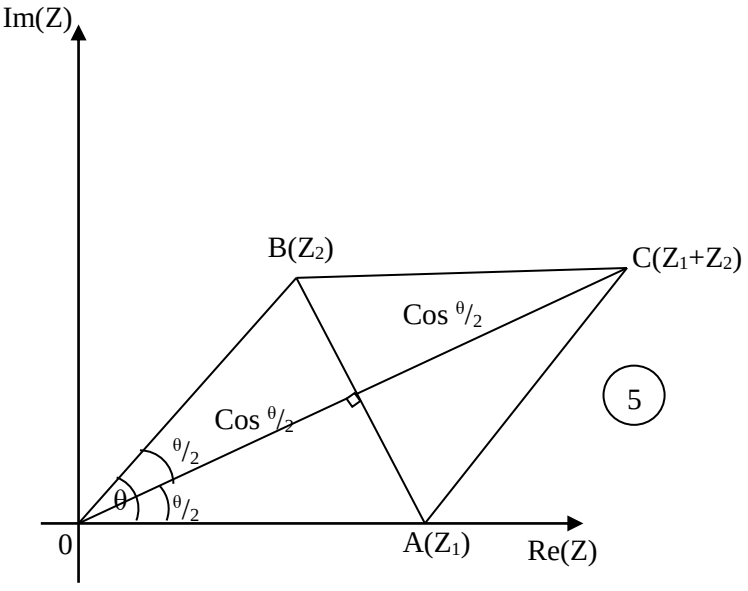
$$C^{-1}(CX) = C^{-1}(AB) \quad (5)$$

$$(C^{-1}C) \times \begin{pmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ 6 & 3 \end{pmatrix} \quad (5)$$

$$IX = \begin{pmatrix} 0-9 & -\frac{5}{2}-\frac{9}{2} \\ 0+12 & 5+6 \end{pmatrix} \quad (5)$$

$$X = \begin{pmatrix} -9 & -7 \\ 12 & 11 \end{pmatrix} \quad (5)$$

(b)



5

C குறிக்கும் சிக்கல் எண் $Z_1 + Z_2$ 'நிறுவல் இணைகரம் OA CB ஒரு சாய்சதுரம். (5)

$\therefore \hat{AOC} = \theta/2, \therefore OC = 2\cos\theta/2$ (5)

$|Z_1 + Z_2| = OC \ \arg|Z_1 + Z_2| = \theta/2$ (5)

$= 2\cos\theta/2$ (5)

$\therefore Z_1 + Z_2 = 2Cos\theta/2\left(Cos\theta/2 + iSin\theta/2\right)$ (5)

$|Z_1 + Z_2| = 2Cos\theta/2$ (5)

$|Z_1 + Z_2|_{\max} = 2 \times 1$

$= 2; \quad Cos\theta/2 = 1$ (5)

$\theta/2 = 0$

$\theta = 0$ (5)

அப்போது $Z_2 = Cos\theta + iSiin\theta$ (5)

$= 1$

$Z = Z_1 + Z_2$

$\Rightarrow x + iy = (1 + \cos\theta) + iSin\theta$ (5)

[மெய்] $\Rightarrow x = 1 + Cos\theta$

$x - 1 = Cos\theta$ .....(1) (5)

[கற்பனை] $\Rightarrow y = Sin\theta$ .....(2) (5)

$(1)^2 + (2)^2 \Rightarrow (x-1)^2 + y^2 = 1$ (5)

$\therefore$ Z இன் ஒழுக்கு (1,0) ஐ மையமாகவும் 1 அலகை ஆரையாகவும் கொண்டவட்டமாகும். (5)

14. (a) $\frac{\tan x}{dx} = \lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h}$ (5)

$$= \lim_{h\to 0} \frac{\frac{Sin(x+h)}{Cos(x+h)} - \frac{Sinx}{Cosx}}{h}$$

$$= \lim_{h\to 0} \frac{Sin(x+h)Cosx - SinxCos(x+h)}{h(Cos(x+h)Cosx}$$

$$= \frac{\lim_{h\to 0}\left(\frac{Sinh}{h}\right)}{\lim_{h\to 0}\left(Cos(x+h)Cosx\right)}$$ (5)

$$= \frac{1}{Cos^2 x} = Sec^2 x$$

$\tan^{-1} x = \theta$

$x = \tan\theta$

$$\frac{dx}{d\theta} = Sec^2\theta \qquad (5)$$

$$\frac{d\theta}{dx} = \frac{1}{Sec^2 x}$$

$$\frac{d\theta}{dx} = \frac{1}{(1+\tan 2\theta)} \qquad (5)$$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{(1+x^2)}$$

$$\frac{d(\tan^{-1} x)}{dy} = \frac{d(\tan^{-1} x)}{dx} \times \frac{dx}{dy} \qquad (5)$$

$$\frac{d(\tan^{-1} x)}{dy} = \frac{1}{(1+x^2)}\frac{dx}{dy} \qquad (5)$$

$$\frac{dy}{d(\tan^{-1} x)} = (1+x^2)\frac{dx}{dy}$$

$$\tan^{-1} x = \theta$$

$$x = \tan^{-1}\theta$$

$$y = \frac{\tan\theta}{\sqrt{1+\tan\theta}} + Cos\left[2\tan^{-1}\left(\frac{\sqrt{1+\tan\theta}-1}{\tan\theta}\right)\right]$$

$$= \frac{Sin\theta/Cos\theta}{1/Cos\theta} + Cos\left[2\tan^{-1}\left(\frac{1/Cos\theta - 1}{Sin\theta/Cos\theta}\right)\right] \qquad (5)$$

$$= Sin\theta + Cos(2\tan^{-1}\left(\tan\theta/2\right)$$

$$= Sin\theta + Cos\theta \qquad (5)$$

$$dy/d\theta = Cos\theta - Sin\theta \qquad (5)$$

$$dy/d\theta = \sqrt{2}\left(1/\sqrt{2}\,Cos\theta - 1/\sqrt{2}\,Sin\theta\right)$$

$$= \sqrt{2}\left(Cos\,\pi/4\,Cos\theta - Sin\pi/4\,Sin\theta\right)$$

$$= \sqrt{2}Cos\left(\pi/4 + \theta\right) \qquad (5)$$

$$dy/d(\tan^{-1} x) = \sqrt{2}Cos\left(\pi/4 + tam^{-1}x\right)$$

(b) $f(x) = \dfrac{1+2x}{x(x+1)} = y$

$x = O', x = (-1) \Rightarrow$ நிலைக்குத்து அணுகு கோடுகள் (5)

$$f'(x) = \frac{(2x+1)(2x+1) + 2(x^2+x)}{x^2(x+1)^2} \qquad (5)$$

$$= \frac{-(4x^2+4x+1)-2x^2-2x}{x^2(x+1)^2}$$

$$= \frac{-(2x^2+2x+1)}{x^2(x+1)^2}$$

$2x^2+2x+1 \Rightarrow$

$O = (2)^2 - 4\times 2\times 1$

$< O = \frac{-2(x^2+x+\frac{1}{2})}{x^2(x+1)^2} = \frac{-2(x+\frac{1}{2})^2+\frac{1}{4}}{x^2(x+1)^2}$ (5)

கற்பணைத்தீர்வு

$(2x^2+2x+1) \neq 0$

∴ திரும்பற்புள்ளிகள் இல்லை

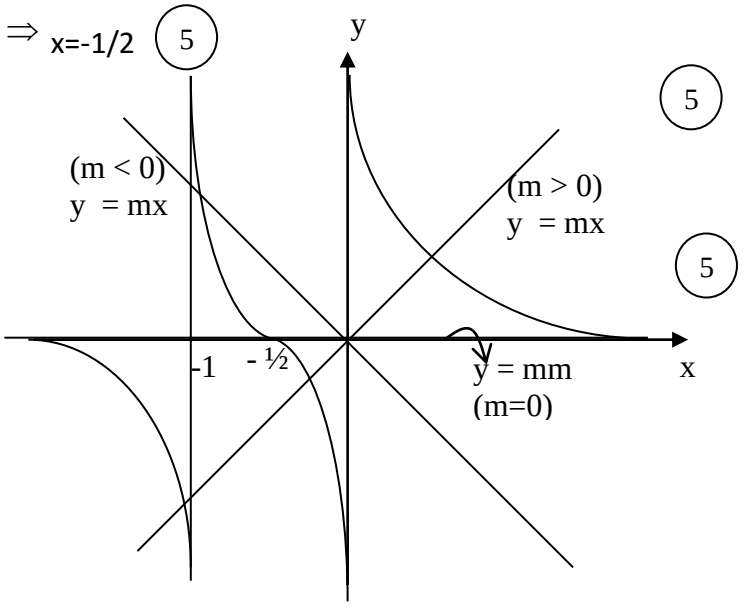
	-1	0	
x > (-1)	-1< x < 0	x > 0	10
f'(x) < 0	f'(x) < 0	f'(x) < 0	

$$\lim_{x\to\pm\alpha} \frac{1+2x}{x(x+1)}$$

$$\lim_{x\to\pm\alpha} \frac{1/x^2 + 2/x}{(1+1/x)} = 0$$

$y = 0 \Rightarrow$ கிடை அணுக்கோடு

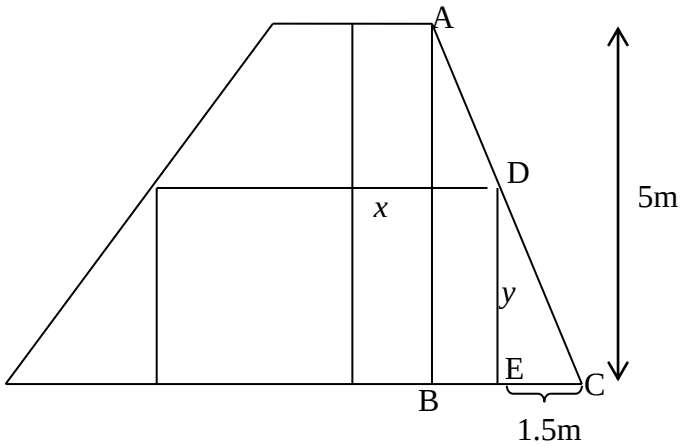
$y = 0 \Rightarrow$ x=-1/2 (5)



i. $m > 0$ ஆகும் போது $y - mx$, $y = f(x)$ ஆகிய வரைபுகள் மூன்று புள்ளிகளின் சந்திப்பதனால் மூன்று மெய்மூலங்களை உடையன. (5)

ii. $m \leq 0$ ஆகும் போது $y = mm$, $y = f(x)$ ஆகிய வரைபுகள் ஒரு புள்ளியில் சந்திப்பதனால் ஒரு மெய்முலத்தை மட்டும் கொண்டிருக்கும். (5)

(c)



$\Delta ABC /// \Delta DEC$

$$\frac{y}{5} = \frac{1.5 - x}{1}$$

$$g = 5(1.5 - x) \Rightarrow x = (1.5 - {}^{y}\!/_{5}) \quad (5)$$

$$V = (\pi x^2)y$$

$$= \pi \frac{(7.5 - y)^2}{5^2} y$$

$$= ({}^{\pi}\!/_{5^2})y(y - 7.5)^2 \quad (5)$$

$$\frac{dV}{dy} = {}^{\pi}\!/_{25}\left((y - 7.5)^2 + 2(y - 7.5) \times (1)y\right) \quad (5)$$

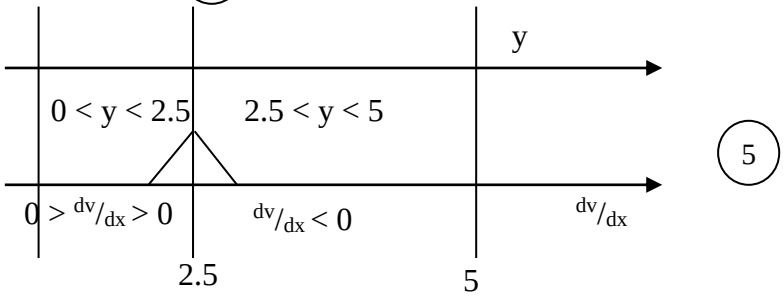
$$\frac{dV}{dy} = {}^{\pi}\!/_{25}(y - 7.5)(3y - 7.5)$$

$$= {}^{3\pi}\!/_{25}(y - 7.5)(y - 2.5) \quad (5)$$

y=7.5, 2.5

But $0 < y < 5$ (5)

$\therefore y = 2.5$ (5)



(5)

y = 2.5 m ஆகும் போது கனவளவு உயர்வாக இருக்கும்.
∴ மிகக்கூடிய கனவளவை உடைய உருளையின் நீளம் 2.5m க்கும் (5)

15. $\frac{x^2-3x+4}{(x-1)^2(x^2+1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-1} + \frac{D}{(x-1)^2}$ (5)

$(x^3 - x^2 + x - 1)$ $(x^3 - 2x^2 + 1)$

$C(x-1)(x^2+1) + D(x^2-1) + Ax + B(x-1)$

$\equiv x^2 - 3x + 4$

$x = 1 \Rightarrow 2D = 2$

$D = 1$ (5)

x^3 $C + A = 0$ ……..(1)

x^2 $-C + D + B - 2A = 1$

$2A + C - B = 0$

x $-3 = C + A - 2B$ ……..(2)

மாறிலி 4 = - C +D + B

B - C =3 …….(3)

(1),(2) → - 2B = -3

$B = (3/2), C = (3/2), A = 3/2$ (5)

$\frac{x^2-3x+4}{(x-1)^2(x^2+1)} = \frac{3(x+1)}{2(x^2+1)} - \frac{3}{2(x-1)} + \frac{1}{(x-1)^2}$

$\int \frac{x^2-3x+4}{(x-1)^2(x^2+1)} dx$

$\int \frac{3(x+1)}{2(x^2+1)} - \frac{3}{2(x-1)} + \frac{1}{(x-1)^2} dx$ (5)

$= 3/2 \int \frac{x}{(x^2+1)} + \frac{1}{(x^2-1)} dx - \int \frac{3}{2(x-1)} + \frac{1}{(x-1)^2} dx$

$= 3/2 \int \frac{2x}{2(x^2+1)} + \frac{1}{(x^2-1)} dx - 3/2 \int \frac{1}{(x-1)} dx + \frac{1}{(x-1)^2} dx$

$= 3/2 (\ln(x^2+1) + \tan^{-1}(x)) - 3/2 \ln(x-1) + C$ (5)

c-தொகையீட்டு மாறிலி (5)

(b) x = a-y x→a , y→0

dx = -dy x→0 , y→a (5)

$\int_0^a f(x)dx = \int_a^0 f(a-y)(-dy)$

(5)

$$= \int_0^a f(a-y)dy$$

$$= \int_0^a f(a-x)dx$$

$$\int_0^{\Pi} \frac{x\tan x}{ssex+\tan x}dx = \int_a^{\Pi} \frac{x\sin x/\cos x}{1/\cos x + \sin x/\cos x}dx$$

$$I = \int_a^{\Pi} \frac{x\sin x}{1+\sin x}dx.....(1) \quad (5)$$

$$I = \int_a^{\Pi} \frac{(\Pi-x)\sin(\Pi-x)}{1+\sin(\Pi-x)}dx \quad (5)$$

$$I = \int_a^{\Pi} \frac{(\Pi-x)\sin x}{1+\sin x}dx.....(2)$$

$$(1)+(2)$$

$$2I = \int_a^{\Pi} \frac{\Pi\sin x}{1+\sin x}dx \quad (5)$$

$$2I = \int_a^{\Pi} \frac{\Pi(1+\sin x)}{1+\sin x}dx$$

$$2I = \int_0^{\Pi} \Pi dx - \Pi\int_a^{\Pi} \frac{1}{1+\sin x}dx$$

$$2I = \Pi x|_0^{\Pi} - \Pi\int_a^{\Pi} \frac{1}{1+\cos(\Pi/2 - x)}dx \quad (5)$$

$$2I = \Pi(\Pi-0) - \Pi\int_a^{\Pi} \frac{1}{2\cos^2(\Pi/4 - x/2)}dx$$

$$2I = \Pi^2 - \frac{\Pi}{2}\int_a^{\Pi} sex^2(\Pi/4 - x/2)dx$$

$$2I = \Pi^2 - \frac{\Pi}{2}\tan(\Pi/4 - x/2)|_0^{\Pi} \quad (5)$$

$$2I = \Pi^2 - \frac{\Pi}{2}[\tan(\Pi/4 - \Pi/2) - \tan(\Pi/4)$$

$$2I = \Pi^2 - \frac{\Pi}{2}(-1-1) \quad (5)$$

$$I = \Pi(\Pi/2 - 1)$$

(c)

$$A = \int_{-1}^{1} \frac{x^2}{1+e^x} dx$$

x = -y x→1 , y→-1

dx = -dy x→-1 , y→1 (5)

$$A = \int_{1}^{-1} \frac{(-y)^2}{1+e^{-y}}(-dy) \quad (5)$$

$$A = \int_{-1}^{1} \frac{y^2}{1+e^{-y}} dy$$

$$A = \int_{-1}^{1} \frac{x^2}{1+e^x} dx$$

A = B

$$A + B = \int_{-1}^{1} \frac{x^2}{1+e^x} dx + \int_{-1}^{1} \frac{x^2}{1+1/e^x} dx \quad (5)$$

$$A + B = \int_{-1}^{1} \frac{x^2}{1+e^x} dx + \int_{-1}^{1} \frac{e^x x^2}{1+e^x} dx$$

$$A + B = \int_{-1}^{1} \frac{(1+e^x)x^2}{(1+e^x)} dx \quad (5)$$

$$A + B = \int_{-1}^{1} x^2 dx$$

$$A + B = \frac{x^3}{3} \Big|_{-1}^{1} \quad (5)$$

$$A + B = [1/3 - (-1/3)]$$

$$A + B = 2/3 \quad (5)$$

$$A = B = 1/3 \quad (5)$$

(d)

$$\int_{0}^{1} x \ln(1 + x/2) dx$$

$= \frac{x^2}{2}\ln(1+x/2)\,|_0^1 - \int_0^1 \frac{x^2}{2}(1/2)\frac{1}{1+x/2}dx$ (5)

$= 1/2\ln(3/2) - \frac{1}{2}\int_0^1 \frac{x^2}{2+x}dx$ (5)

$\frac{x^2}{2+x} \equiv Ax + B + \frac{C}{2+X}$ (5)

$(Ax+B)(2+x)+C \equiv x^2$

$A=1, B=-2, C=4$ (5)

$= 1/2\ln(3/2) - \frac{1}{2}\int_0^1 [(x-2)+\frac{4}{2+x}]dx$ (5)

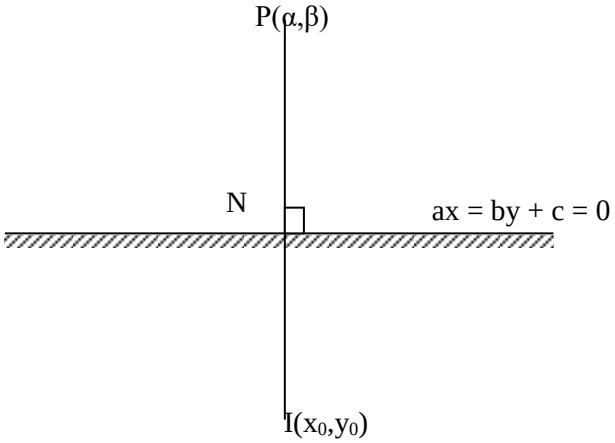
$= 1/2\ln(3/2) - \frac{1}{2}[(\frac{x^2}{2}-2x)+4\ln(2+x)]\,|_0^1$ (5)

$= 1/2\ln(3/2) - \frac{1}{2}\ln 3/2 + 3/4$

$= -3/2\ln(3/2) + 3/4$ (5)

$a = 3/4, b = 3/2$ (5)

16.



විම්පම් $I \equiv (x_0, y_0)$ එන්ග

$\left(\frac{y_0-\beta}{x_0-\alpha}\right)\left(-\frac{a}{b}\right) = -1$ (5)

$\frac{y_0-\beta}{b} = \frac{x_0-\alpha}{a} = t$ එන්ග (5)

$x_0 = \alpha + at$

$y_0 = \beta + at$ (5)

$N \equiv \left\{\frac{x_0+\alpha}{2}\right\} + b\left\{\frac{y_0+\beta}{2}\right\} + C = 0$ ආනතු $ax+by+c=0$ එන්නුම් කෝට්ටිල් இருப்பதால்.

$$a\left\{\frac{x_0+\alpha}{2}\right\}+b\left\{\frac{y_0+\beta}{2}\right\}+C=0 \quad (5)$$

$$a\left\{\frac{\alpha+at+\alpha}{2}\right\}+b\left\{\frac{\beta+bt+\beta}{2}\right\}+C=0$$

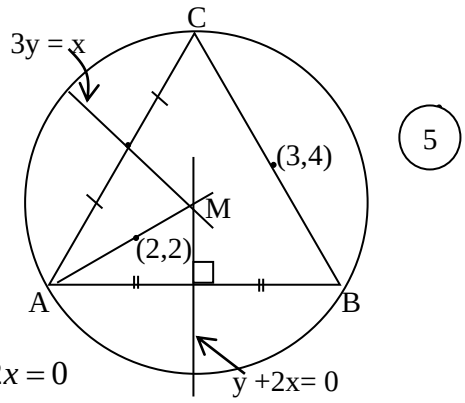
$$a\left\{\alpha\frac{at}{2}\right\}+b\left\{\beta\frac{bt}{2}\right\}+C=0$$

$$\frac{t}{2}\left\{a^2+b^2\right\}=-a\alpha-b\beta-C$$

$$t=\frac{-2(a\alpha+b\beta+C)}{a^2+b^2} \quad (5)$$

∴ விம்பம் I இன் ஆள்கூறுகள்

$$I\equiv\left\{\alpha-\frac{2a(a\alpha+b\beta+c}{a^2+b^2}\beta-\frac{2b(a\alpha+b\beta+c}{a^2+b^2}\right\} \quad (5)$$



(5)

$$y+2x=0$$

$$\underline{3y=3}$$

$$x=0, y=0$$

$$\therefore N\equiv(0,0) \quad (5)$$

$$AN \Rightarrow \frac{y-0}{x-0}=\frac{2-0}{2-0}$$

$$AN \Rightarrow y=x \quad (5)$$

let $A\equiv(x_0, y_0)$

but $x_0 \equiv y_0 = t$

$$\therefore A\equiv(t,t) \quad (5)$$

B ஆனது y + 2x = 0 பற்றி A யின் ஆடிவிம்பம் என்பதால்

$$B\equiv\left\{t-\frac{2(2)[2\times t+1\times t+0}{2^2+1^2}, t-\frac{2(1)[2t+t}{2^2+1^2}\right\} \quad (5)$$

$$B\equiv\left\{-\frac{7t}{5}, t-\frac{t}{5}\right\} \quad (5)$$

புள்ளி C ஆனது x-3y = 0 பற்றி புள்ளி A இன் ஆடிவிம்பம் ஆதலால்

$$C \equiv \left\{ t - \frac{2(1)(1\times t - 3t)}{1^2 + 3^2}, t - \frac{2(-3)(1\times t - 3t)}{1^2 + 3^2} \right\}$$ (5)

$$C \equiv \left\{ \frac{7t}{5}, \frac{-t}{5} \right\}$$ (5)

$$M_{BC} \equiv \left\{ \frac{-t/5 - \left(-t/5\right)}{7t/5 - \left(-7t/5\right)} = \frac{4 - \left(-t/5\right)}{3 - 7t/5} \right\}$$ (5)

$$0 = 4 + t/5$$

$$4 + t/5 = 0$$

$$t = -20$$ (5)

$A \equiv (-20,-20)$
$B \equiv (28,4)$
$C \equiv (-28,4)$ (15)

(b) சுற்றுவட்ட மையம் $N \equiv (0,0)$

ஆரை (r) $= \sqrt{[0-(-20)]^2 + [0-(-20]^2}$ (5)

$= \sqrt{20^2 + 20^2}$

$r = 20\sqrt{2}$ (5)

$\therefore$ சுற்றுவட்டத்தின் சமன்பாடு $\Rightarrow (x-0)^2 + (y-0)^2 = (20\sqrt{2})^2$

$x^2 + y^2 = 800$

$x^2 + y^2 - 800 = 0$ (5)

(c) A,N இனூடு செல்லும் வட்டத்தின் சமன்பாடு $x^2 + y^2 + 2gx + 2fy + C = 0$ என்க

$(0,0) \Rightarrow C = 0$ (5)

$(-20,-20) \Rightarrow 400 + 400 - 40g - 40f + c = 0$

$g + f = 20$ (5)

$g = t'$ என்க

$\therefore\ f = 20 - t'$ (5)

$\Rightarrow x^2 + y^2 + 2tx + 2(20 - t')y + 0 = 0$ (5)

$(t'-t) \Rightarrow x^2 + y^2 + 2tx + (40 - 2t')y = 0$

AN ஐ நாணாக கொண்ட வட்டங்களை விட AN ஐ விட்டமாக கொண்ட வட்டமே மிகச்சிறியது.

ஆரை $r' = \sqrt{t^2 - (20-t)^2}$

$= \sqrt{40t - 400}$ (5)

$$\sqrt{40t-400}=\frac{20\sqrt{2}}{2}$$

$40t-400=200$

$40t=600$

$t=15$ (5)

$\therefore$ மிகச்சிறி a ஆரையைக் கொண்ட வட்டத்தின் சமன்பாடு $x^2+y^2+30x+10y=0$ (5)

17. (a) $$L.H.S=\frac{Cos2x+\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}Sinx}{(1+Sin2x)+\frac{1}{\sqrt{2}}Cosx+\frac{1}{\sqrt{2}}Sinx}$$ (5)

$$=\frac{(Cos2x-Sin2x)+\frac{1}{\sqrt{2}}(\cos x-Sinx)}{(Cos2x+Sin2x+2Sinx.\cos x)+\frac{1}{\sqrt{2}}(Sinx+Cosx)}$$ (5) (5)

$$=\frac{(Cosx-Sinx)\left(\cos x+Sinx+\frac{1}{\sqrt{2}}\right)}{(Cosx+Sinx)+(Sosx+Sinx+\frac{1}{\sqrt{2}})}$$ (5)

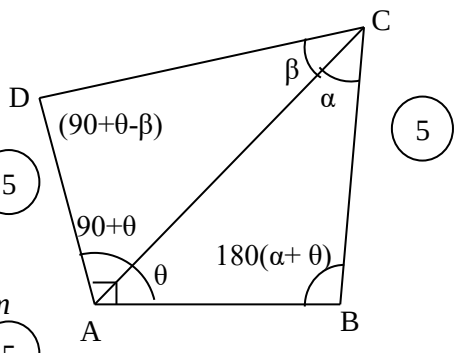
$$=\left(\frac{Cosx-Sinx}{Cosx+Sinx}\right)$$

(b) $$\frac{a}{SinA}=\frac{b}{SinB}=\frac{c}{SinC}$$ (5)

$$\frac{a}{SinA}=\frac{b}{SinB}=\frac{c}{SinC}=k$$

Δ ABC இல் சைன் விதிப்படி

$$\frac{l}{SinA}=\frac{AC}{Sin[180-(\alpha+\theta)]}\Rightarrow AC=\frac{Sin(\alpha+\theta)}{Sin\alpha}l$$ (5) (5)



(5)

Δ ADC இல் சைன் விதிப்படி

$$\frac{A}{Sin\beta}=\frac{AC}{Sin[90-(\theta-\beta)]}\Rightarrow AC=\frac{Cos(\theta-\beta)}{Sin\beta}m$$ (5) (5)

$$\therefore AC=\left[\frac{Sin(\alpha+\theta)}{Sin\alpha}\right]l=\left(\frac{Cos(\theta-\beta)}{Sin\beta}\right)m$$ (5)

$$\frac{(Sin\alpha Cos\theta+Cos\alpha Sin\theta)}{Sin\alpha}l=\left(\frac{Cos\theta Cos\beta+Sin\theta Sin\beta}{Sin\beta}\right)m$$

$$(Cos\theta+\cot\alpha Sin\theta)l=(Cot\beta Cos\theta+Sin\theta)m$$

$$(Cot\theta+\cot\alpha)l=(Cot\beta Cot\theta+1)m$$ (5)

$$Cot\theta[l-m\cot\beta]=m-l\cot\alpha$$ (5)

$$\therefore Cot\theta=\frac{m-l\cot\alpha}{l-m\cot\beta}$$

(c) $f(x)=4(Sin^4x+Cos^4x)$

(5)

$$= 4\left[(Sin^2 x + Cos^2 x)^2 - 2Sin^2 x Cos^2 x\right]$$

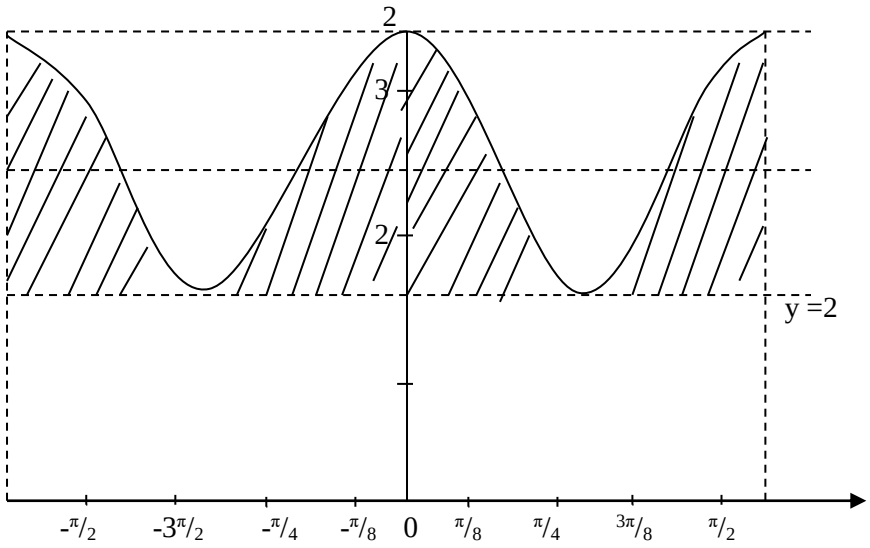
$$= 4\left[1 - \frac{1}{2} \times (Sin2x)^2\right] \quad (5)$$

$$= 4 - 2Sin^2 2x \quad (5)$$

$f(x) = 3 + Cos4x$

x	$-\pi/2$	$-3\pi/8$	$-\pi/4$	$-\pi/8$	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
$4x$	-2π				0		π		2π
Cos $4x$	1	0	-1	0	1	0	-1	0	1
3+ Cos$4x$	4	3	2	3	4	3	2	3	4

(20)



(15)

$y = f(x), x = \pm\pi/2, y = 2$ இனால் வரைபுள்ள பரப்பளவு = 0

$$A = \int_{-\pi/2}^{\pi/2} (3 + Cos4x)dx - 2 \times \left[\pi/2 - \left(-\pi/2\right)\right] \quad (10)$$

$$= 3\int_{-\pi/2}^{\pi/2} dx + \int_{-\pi/2}^{\pi/2} Cos4x dx - 2\pi \quad (5)$$

$$= 3x\Big|_{-\pi/2}^{\pi/2} + \frac{Sin4x}{4}\Big|_{-\pi/2}^{\pi/2} - 2\pi \quad (5)$$

$$A = 3\Pi + (0+0) - 2\Pi \quad (5)$$

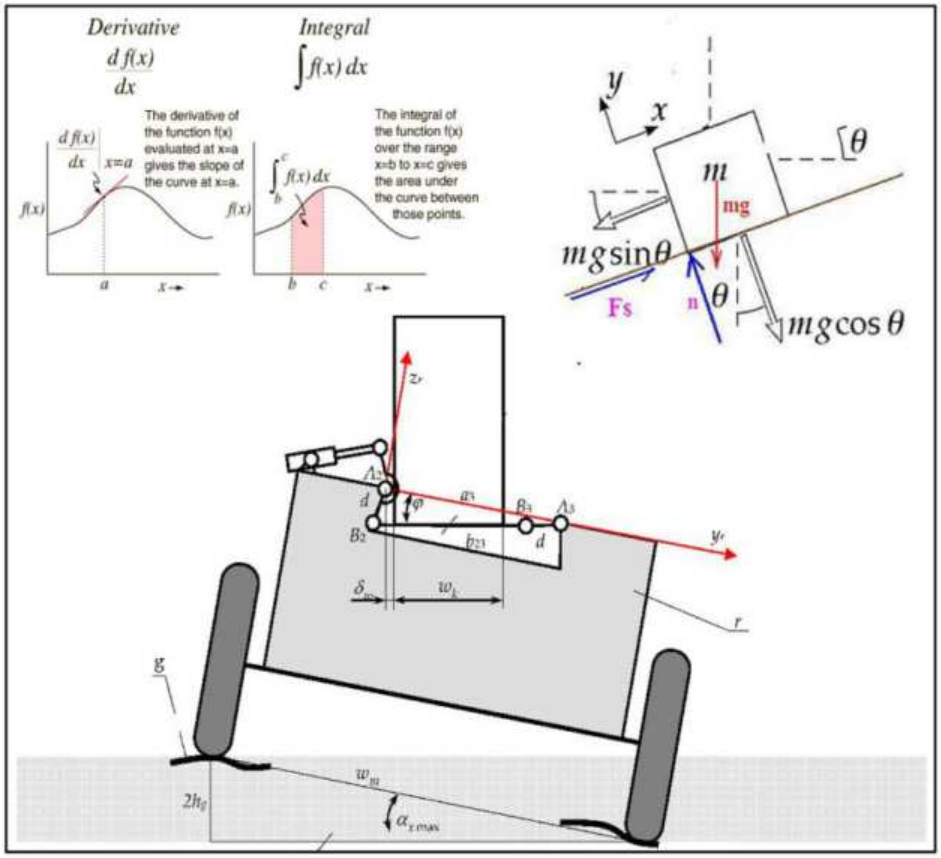
$$A = \Pi \quad (5)$$

மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள்
நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 7 வது

முன்னோடிப் பரீட்சை - 2016

10(II) - இணைந்தகணிதம் II

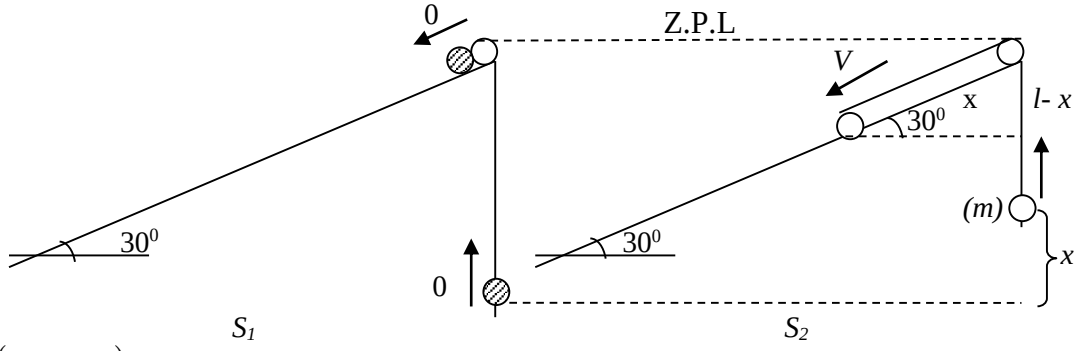
விடைகள் (புள்ளியிடும் திட்டம்)



Prepared By
P.Senthilnathan *B.Sc. Dip in Edu*

விடை

(1)



$(S_1 \to S_2)$ சக்திகாப்பு தத்துவம்

$-mgl = -4mg(x\sin 30) - mg(l-x) + \frac{1}{2} \times 4m \times V^2 + \frac{1}{2} mV^2$ (15)

$\Rightarrow V^2 = \frac{2gx}{5}$ (5)

t குறித்து வகையிட

$2V\frac{dV}{dt} = \frac{2g}{5}\frac{dx}{dt}$

$\frac{dV}{dt} = \frac{g}{5}, \because \frac{dx}{dt} = V$ (5)

$\therefore$ தொகுதியின் ஆர்முடுகல் $= \frac{g}{5}$

---

(2) $V = u + at$

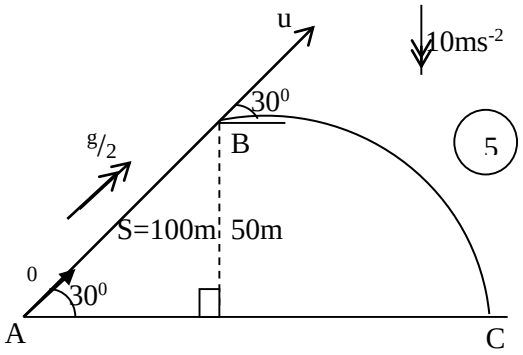
$\nearrow u = 0 + \frac{9}{2} + \frac{20}{3}$

$u = 30ms^{-1}$ (5)

$v^2 = u^2 + 2as$

$\nearrow \quad 900 = 0 + 2 \times \frac{9}{2} \times s$

$S = 100m$ (5)



புவியீர்ப்பின் கீழ் இயக்கத்தில் $\nearrow$

$S = ut + \frac{1}{2}at^2$

$(B \to C) \uparrow -50 = 30\sin 30t - \frac{1}{2} \times 10t^2$ (5)

$5t^2 - 15t - 50 = 0$

$t^2 - 3t - 10 = 0$

$(t-5)(t+2) = 0$

$t = 5s, \because t > 0$ (5)

---

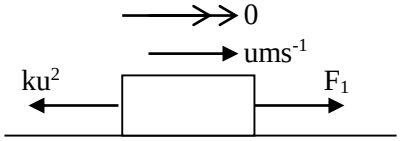
(3) $F = m\underline{a}$

$\rightarrow F_1 - ku^2 = Mo$

$F_1 = ku^2$ (5)

$p = F \times V$

$p = ku^2 \times u$



$p = ku^3$ (5) ….(1)

$\underline{F} = m\underline{a}$

$F_2 - kv^2 - Mg\sin\alpha = Mo$ (5)

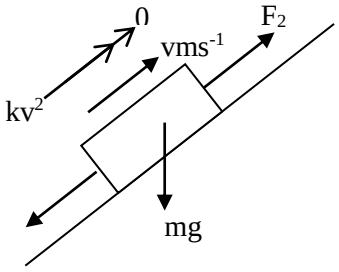
$F_2 = kv^2 - Mg\sin\alpha$

$p = F \times V$

$p = (kV^2 + Mg\sin\alpha)V$ ….(2) (5)

$(1) = (2) \Rightarrow ku^3 + MgV\sin\alpha$

$K = \dfrac{MgVSiin\alpha}{u^3 - V^3}$ (5)



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(4) உ.கா.விதி

$\rightarrow m\times 3 - m\times 5 = -mu + mv$ (1)

$v - u = -2$ ……..(1)

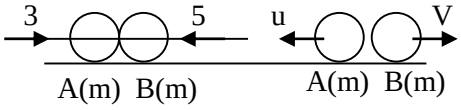
நி. ப. விதி $u + v = e(3+5)$ (5)

$v + u = 8e$

$(1) + (2) \Rightarrow V = \dfrac{2(4e-1)}{2}$ (5)

$V = 4e - 1 > 0$ (5)

$\Rightarrow e > ¼$



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(5) $\underline{a} = \underline{i} + 2\underline{j}$

$\underline{b} = 2\underline{i} - \underline{j}$

$\overrightarrow{OC} = \underline{c} = \lambda\underline{a}$

$= \lambda\underline{i} + 2\lambda\underline{j}$

$\overrightarrow{OD} = \underline{d} = \mu\underline{b} = 2\mu\underline{i} - \mu\underline{j}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\overrightarrow{AB} = \underline{i} - 3\underline{j}$

$\overrightarrow{CD} = \underline{d} - \underline{c}$ (5)

$\overrightarrow{CD} = (2\mu - \lambda) - (2\lambda + \mu)\underline{j}$

$\left[\overrightarrow{CD}\right] = \sqrt{(2\mu - d)^2 + (2\lambda + \mu)^2} = 2\sqrt{10}$

$(2\mu - d)^2 + (2\lambda + \mu)^2 = 40$

$5(\mu^2 + \lambda^2) = 10$

$\lambda^2 + \mu^2 = 8$ ……(1) (5)

$AB \perp CD$

$\overrightarrow{AB} - \overrightarrow{CD} = 0$

$(\underline{i} - 3\underline{j}).[(2\mu - \lambda)\underline{i} - (2\lambda + \mu)\underline{j}] = 0$

$(2\mu - \lambda) + 3(2\lambda + \mu) = 0$

$5(\mu + \lambda) = 0$ (5)

$\mu = -\lambda$ ….(2) ஐ (1) இல் இட

$\lambda^2 + \lambda^2 = 8$

$\lambda^2 = 4$

$\lambda = \pm 2$

$\lambda = 2$ எனின் (1) $\Rightarrow$ $\mu = -2$

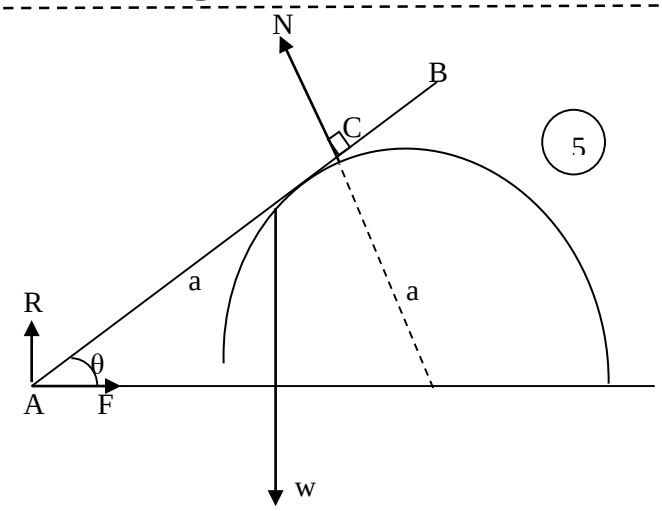
$\lambda = -2$ எனின் (1) $\Rightarrow$ $\mu = 2$ } 5

But $\lambda > \mu$

$\therefore \lambda = 2, \quad \mu = -2$ 5

---

(6)



5

$Sin\theta = 3/5$

↶ A $Na\cot\theta = W \times a\cos\theta$

$N = WSin\theta$

$N = \frac{3}{5}W$ 5

$\uparrow R + N\cos\theta = W$

$R = W - \frac{3W}{5} \times \frac{4}{5}$

$R = \frac{13W}{25}$ 5

$\rightarrow F = NSin\theta$

$= \frac{3W}{5} \times \frac{3}{5}$ 5

$F = \frac{9W}{25}$

$F/R \leq \mu$ 5

$\Rightarrow \mu \geq \frac{9}{13}$

---

(7) A, B, C யாவுமளாவிய நிகழ்ச்சிகள்

$A \cup B \cup C = \Omega$

$\Rightarrow P(A \cup B \cup C) = 1$ 5

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) = 1$

$6x + x + x + 0 - y + 0 + 0 = 1$

$\because$ [A, B தம்முள் புறநீங்குபன
A, C தம்முள்புறநீங்குபன]

$y = 8x - 1$ ……..(1) (5)

$P(B \cap C) = y \geq 0,$ $P(B \cap C) \leq P(B) or P(C)$

$\Rightarrow 8x - 1 \geq 0$ (5) $y \leq x$ (5)

$\Rightarrow x \geq 1/8$ $8x - 1 \leq x$

$\therefore 1/8 \leq x \leq 1/7$ (5) $x \leq 1/7$

(8) $P(A|C) \geq P(B|C)$

$\Rightarrow \frac{P(A \cap C)}{P(C)} \geq \frac{P(B \cap C)}{P(C)} \Rightarrow P(A \cap C) \geq P(B \cap C)$ (10)

$P(A|C') \geq P(B|C')$

$\frac{P(A \cap C')}{P(C')} \geq \frac{P(B \cap C')}{P(C')}$

$\Rightarrow P(A) - P(A \cap C) \geq P(B) - P(B \cap C)$ ……..(2) (10)

$(1) + (2) \Rightarrow P(A) \geq P(B)$ (5)

---

(9) $f = kx$ ……..(1) (5)

இடை $\bar{x} = \frac{\sum fx}{\sum f}$ (5)

$= \frac{10f_1 + 20f_2 + 30f_3 + 40f_4 + 50f_5}{f_1 + f_2 + f_3 + f_4 + f_5}$ (10)

$= \frac{(10 \times 10k) + (20 \times 20k) + (30 \times 30k) + (40 \times 40k) + (50 \times 50k)}{10k + 20k + 30k + 40k + 50k}$

$= \frac{100 + 400 + 900 + 1600 + 2500}{150}$

$= \frac{550}{15}$ (5)

$\bar{x} = 33.33$

---

(10) இடை = 8

$\frac{1 + 3 + 8 + 8 + a + b + c + d + e + f}{10} = 8$ (5)

$a + b + c + d + e + f = 60$ …..(1)

ஆகாரம் = 9

எண் 8 இரு முறை உள்ளதால், எண் 9 ஆனது குறைந்தது 3 தடைவையேனும் வரல் வேண்டும். எண்கள் ஏறுவரிசையில் உள்ளதால். (5)

$\therefore a = b = c = 9$ ஆகும்.

$(1) \Rightarrow d + e + f = 33$ …….(1)'

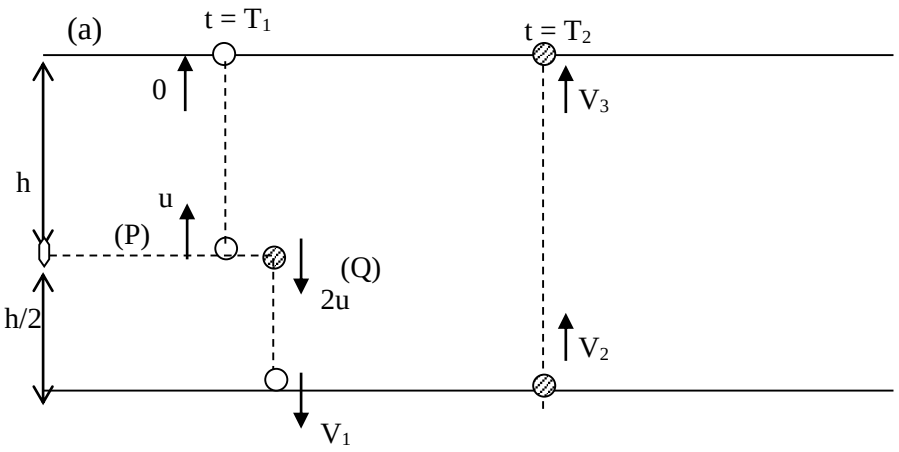
d = 9 எனின் $e = 10, f = 11$ ஆகும். (5)

இல் இருந்து இது சாத்தியமில்லை

$d = 10$ எனின் $e = 1, f = 12$ ஆகும் இது (1)' இற்கு அமைவாகும் (5)

a = b = c = 9, d =10, e =11, f =12 (5)

(11) (a)



Ⓟ $\uparrow v^2 = u^2 + 2as$

$0 = u^2 - 2gh$

$u^2 = 2gh$ (5) .....(1)

Ⓟ $V = u + at$

$\uparrow 0 = u - gt$

$T_1 = u/g$ (5) .....(2)

$v^2 = u^2 + 2as$

$\downarrow V_1^2 = (2u)^2 + 2g \times h/2$

$= 4u^2 + \frac{u^2}{2}$; *from*(1) (5)

$V_1^2 = \frac{9u^2}{2}$

$V_1 = \frac{3u}{\sqrt{2}}$

நிலத்துடன் மோதியவுடன் வேகம்

$V_2 = eV_1 = \frac{1}{\sqrt{3}} \times \frac{3u}{\sqrt{2}} = \sqrt{\frac{3}{2}}u$ (5)

Ⓠ $v^2 = u^2 + 2as$

$\uparrow v_3^2 = v_2^2 - 2g \times \frac{3h}{2}$

$= \frac{3}{2}u^2 - 3gh$ (5)

$= \frac{3}{2} \times 2gh - 3gh$; *from*(1)

$\Rightarrow V_3 = 0$ (5)

$V = u + at$

Ⓠ $\downarrow v_1 = 2u + gt_1$

$gt_1 = \frac{3u}{\sqrt{2}} - 2u$

$t_1 = \frac{u}{g}\left(\frac{3}{\sqrt{2}} - 2\right)$ (5)

$V = u + at$

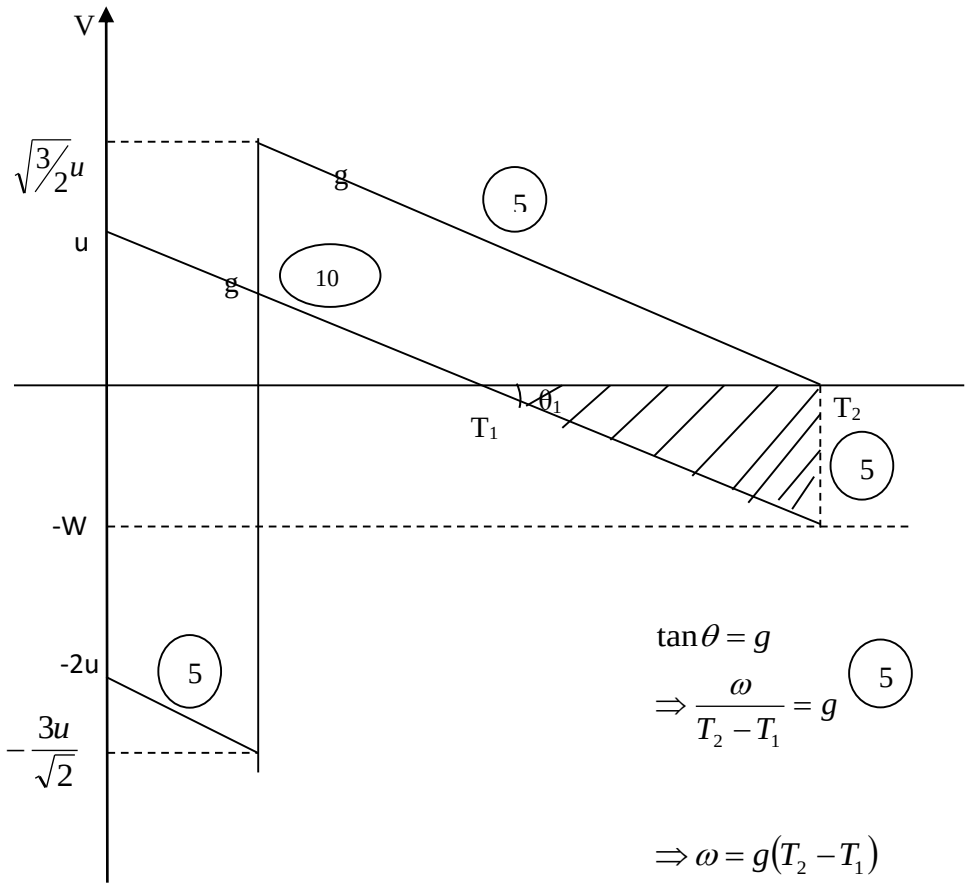
Ⓠ $\uparrow 0 = v_2 - gt_2$

$t_2 = \frac{u}{g}\sqrt{3/2}$ (5)

But $T_2 = t_1 + t_2$

$= \frac{u}{g}\left[\frac{3}{\sqrt{2}} - 2 + \frac{\sqrt{3}}{\sqrt{2}}\right]$

$T_2 = \frac{u}{\sqrt{2}g}\left[3 - 2\sqrt{2} + \sqrt{3}\right]$ (5)



$$\tan\theta = g$$

$$\Rightarrow \frac{\omega}{T_2 - T_1} = g \quad (5)$$

$$\Rightarrow \omega = g(T_2 - T_1)$$

θ ஆனது சீலிங்கை அடையும் போது சீலிங்கிற்கு கீழ்
இன் ஆழம் = நிழற்றப்பட்ட பிரதேசத்தின் பரப்பளவு

$$= \tfrac{1}{2}(T_2 - T_1)\omega \quad (5)$$

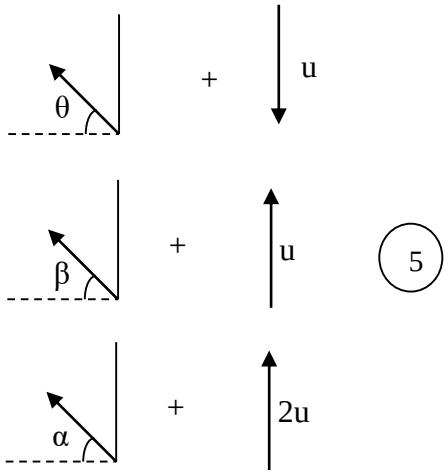
$$= \tfrac{1}{2} g(T_2 - T_1)^2$$

$$= \tfrac{1}{2} g\left[\frac{u}{\sqrt{2}g}\left(3 - 2\sqrt{2} + \sqrt{3}\right) - \frac{u}{g}\right]^2 \quad (5)$$

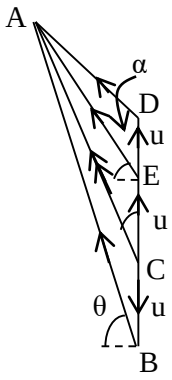
$$= \frac{u^2}{4g}\left(3 - 3\sqrt{2} + \sqrt{3}\right)^2 = \frac{3u^2}{4g}\left(\sqrt{3} - \sqrt{6} + 1\right)$$

85

(b) சைக்கிளோட்டி –C
காற்று - W
பூமி - E
$V_{WE} = V_{WC} + V_{CE}$ (5)



(5)



10 + 10 + 10

(m + n)Cotθ தோற்றத்தினை ΔABD பிரயோகிக்க.

$3\cot(90+\beta) = 2\cot(90+\alpha) - 1\cot(90-\theta)$ 10

$-3\tan\beta = -2\tan\alpha - \tan\theta$ 5

ΔABE (m + n) cotθ தோற்றத்தினை பிரயோகிக்க.

$2\cot\alpha = \cot(90-\theta) - \cot(90+\beta)$ 5

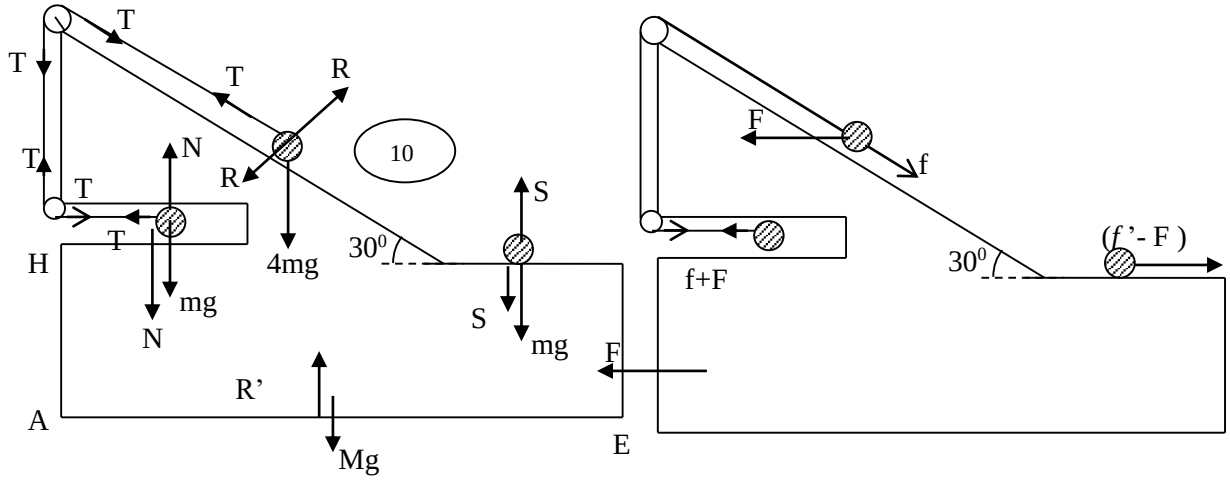
$2\cot\alpha = \tan\theta + \tan\beta$

$$\tan\alpha = \left(\frac{2}{\tan\theta + \tan\beta}\right)$$

$$\alpha = \tan^{-1}\left(\frac{2}{\tan\theta + \tan\beta}\right)$$ 5

65

(12) (a)



10

$\underline{F} = m\underline{a}$

$\searrow 4mg\cos 60 - T = 4m(f - F\cos 30) \Rightarrow 2mg - T = 4m\left(f - \frac{\sqrt{3}}{2}F\right)$ .....(1) 10

$(m) \leftarrow T = m(f + F)$ .....(2) 10

$(m), \rightarrow O = m(f' + F) \Rightarrow f' = F$ .....(3) 10

தொகுதி $0 = MF + m(f + F) + 4m(F - f\cos 30) + m(F - f')$

$$0 = (M + 5m)F + m\left(1 - 4\times\frac{\sqrt{3}}{2}\right)f$$ 15

$\left(2\sqrt{3}-1\right)mf = (M+5m)F$ ……(4)

$(1)+(2) \Rightarrow 2mg = m[5f-(2\sqrt{3}-1)F]$

$5f-(2\sqrt{3}-1)F = 2g$ (5)

$5\dfrac{(M+5m)F}{(2\sqrt{3}-1)m} - \left(2\sqrt{3}-1\right)F = 2g; From(4)$ (5)

$F\left[5(M+5m)-\left(2\sqrt{3}-1\right)^2 m\right] = 2(2\sqrt{3}-1)mg$

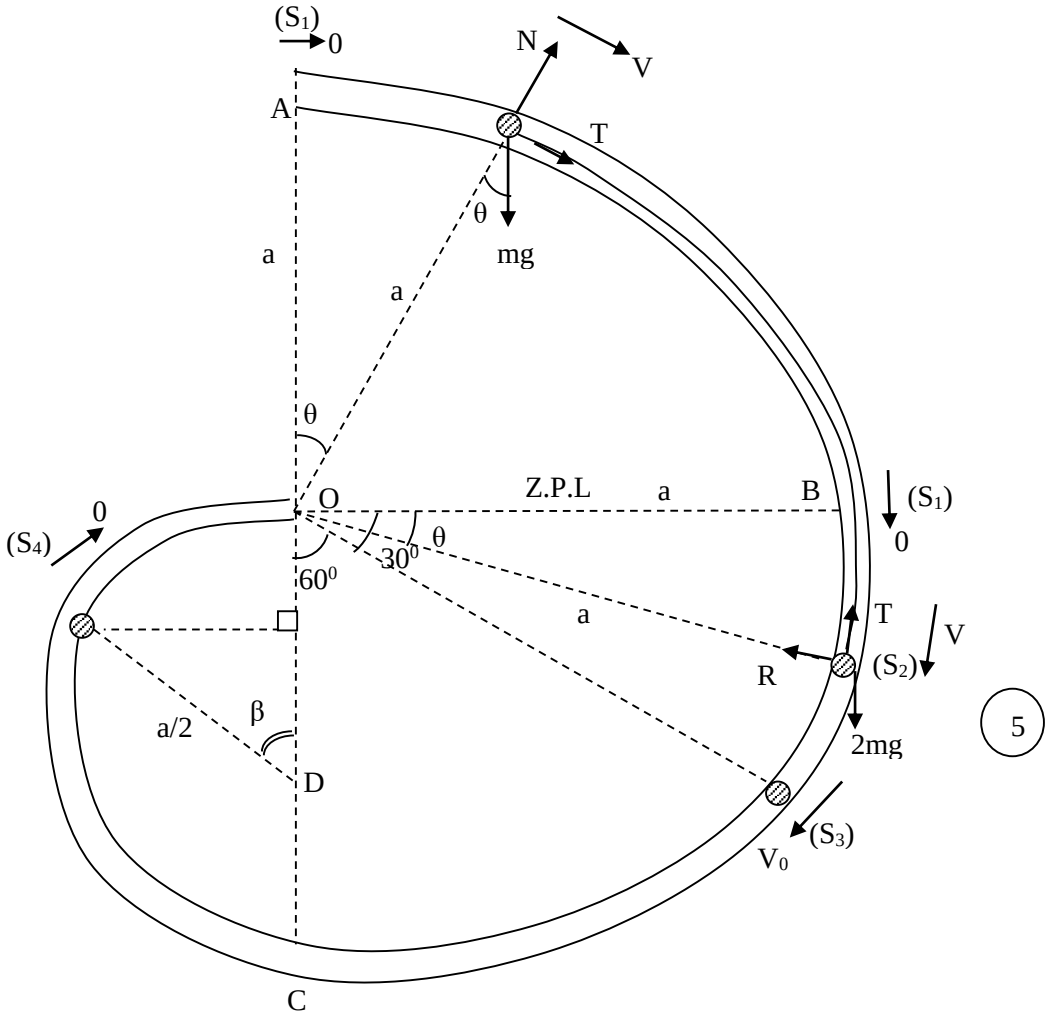
$F = \dfrac{2(2\sqrt{3}-1)mg}{5M+m\left[25-(12+1-4\sqrt{3})\right]}$

$= \dfrac{2(2\sqrt{3}-1)mg}{5M+(12+4\sqrt{3})}$ (5)

$F = \dfrac{2(2\sqrt{3}-1)mg}{5M+4\sqrt{3}\left(\sqrt{3}+1\right)m}$

70

(b)



$(S_1 \to S_2)$ ச. கா. தத்துவம்

$mga = mga\cos\theta - 2mga\sin\theta + \frac{1}{2}\times 2mV^2 + \frac{1}{2}mv^2$ (5) + (5) + (5) + (5)

$2ga(1-\cos\theta+2\sin\theta) = 3V^2$ (5)

$$V^2 = \frac{2ag}{3}\left(1-\cos\theta+2\sin\theta\right) \qquad \ldots\ldots\ldots(1)$$

But $V = a\dot{\theta}$

$$\left(a^2\dot{\theta}^2\right) = \frac{2ag}{3}(1-\cos\theta+2\sin\theta) \qquad (5)$$

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{2g}{3a}(1-\cos\theta+2\sin\theta) \qquad \ldots\ldots\ldots\ldots(1)'$$

$(1)' \Rightarrow \dot{\theta}^2 = \frac{2g}{3a}\left(1-\cos\theta+2\sin\theta\right)$ (5)

t குறித்து வகையிட

$$2\dot{\theta}\ \ddot{\theta} = \frac{2g}{3a}\left(0+\sin\theta+2\cos\theta\right)\dot{\theta}$$

$$a\ddot{\theta} = \frac{g}{3}\left(Sin\theta+2\cos\theta\right)) \qquad (5)\ldots\ldots\ldots(2)$$

$$\underline{F} = m\underline{a}$$

$$\searrow T = \frac{mg}{3}\left(Sin\theta+2\cos\theta\right) - mgSin\theta \qquad (5)$$

$$T = \frac{2mg}{3}\left(Cos\theta - Sin\theta\right) \qquad \ldots\ldots\ldots..(3)$$

$\theta = 30^0$ இல் $V = V_0$ என்க.

$$(1) \Rightarrow V_0^2 = \frac{2ag}{3}\left(1-\frac{\sqrt{3}}{2}+2\times\frac{1}{2}\right) \qquad (5)$$

$$V_0^2 = \frac{2ag}{3}\left(2-\frac{\sqrt{3}}{2}\right)$$

$$V_0 = \sqrt{\frac{2ag}{3}\left(2-\frac{\sqrt{3}}{2}\right)} \qquad (5)\ldots\ldots\ldots..(4)$$

இற்கு $(S_3 \to S_4)$ ச.கா தத்துவம்

$$\frac{1}{2}\times 2mV_0^2 - 2mga\cos 60 = -2mg(a/2 - a/2\cos\beta) \qquad (5) + (5) + (5)$$

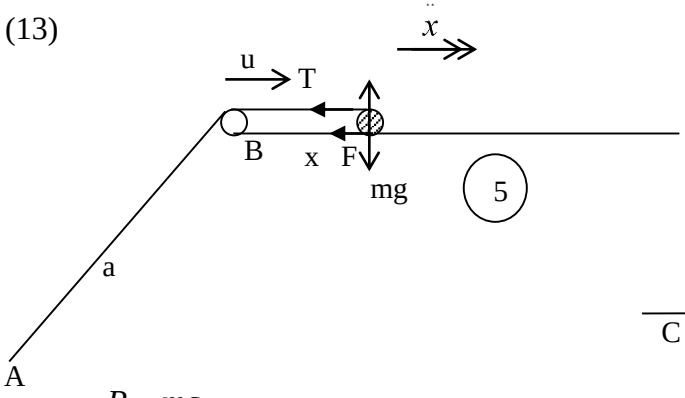
$$V_0^2 - ag = -ag(1-\cos\beta)$$

$$\frac{2ag}{3}(2-\frac{\sqrt{3}}{2}) = ag\cos\beta$$

$$Cos\beta = \frac{4-\sqrt{3}}{3} \qquad (5)$$

$$\beta = Cos^{-1}\left(\frac{4-\sqrt{3}}{3}\right)$$

(13)



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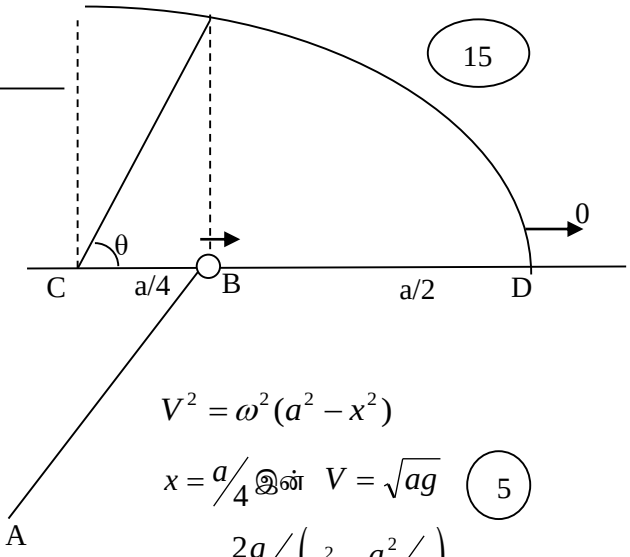


$R = mg$

$F = \mu R = \frac{1}{2}mg$ 5

$T = \frac{2mgx}{a}$ (hooke's law) 5

$\underline{F} = m\underline{a}$

$\rightarrow -T - F = m\ddot{x}.$ 5

$-\frac{2mgx}{a} - \frac{1}{2}mg = m\ddot{x}$ 5

$\ddot{x} = \frac{-2g}{a}(x + a/4)$ 5

$X = x + \frac{a}{4} \Longrightarrow \ddot{X} = \ddot{x}$ 5

$\ddot{X} = -\omega^2 X \quad , \quad \omega^2 = 2g/a$ 5

$X = x + a/4$ 5

y = 0 இல் அலைவு மையம் இருக்கும்

$\Rightarrow x = -a/4$ 5

$V^2 = \omega^2(a^2 - x^2)$

$x = a/4$ இன் $V = \sqrt{ag}$ 5

$ag = \frac{2g}{a}\left(a^2 - a^2/16\right)$

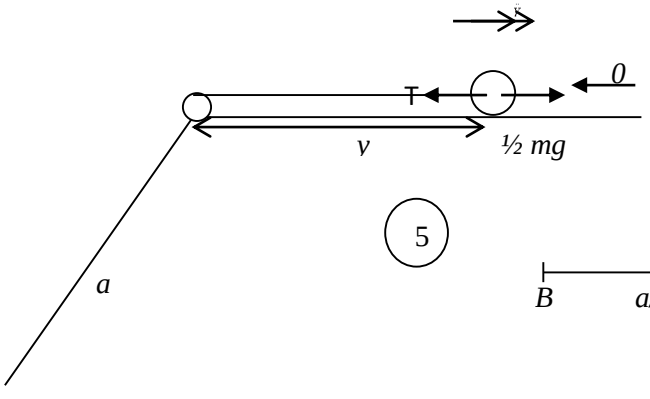
$\frac{a^2}{2} + \frac{a^2}{16} = a_0^2$ 5

$a_0^2 = \frac{9a^2}{16}, a_0 = \frac{3a}{4}$ 5

$\cos = \frac{a/4}{3a/4} = \frac{1}{3}$ 5

$t_1 = \frac{\theta}{\omega}$

$= \sqrt{\frac{a}{2a}}\cos^{-1}\left(\frac{1}{3}\right)$ 5



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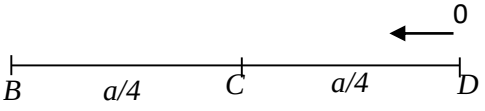


$\underline{F} = m\underline{a}$

$\frac{1}{2}mg - T = m\ddot{y}$ 5

$\frac{1}{2}mg - \frac{2mgy}{a} = m\ddot{y}$ 5

$\ddot{y} = -\frac{2g}{a}\left(y - \frac{a}{4}\right) = 2$ 5

5 $\ddot{Z} = -\omega^2 Z, \quad \omega^2 = \frac{2g}{a}$ 5

$x = \frac{a}{4}$ அலைவு மையம் 5

வீச்சம் = a/4, B யில் F = 0, T = 0 5

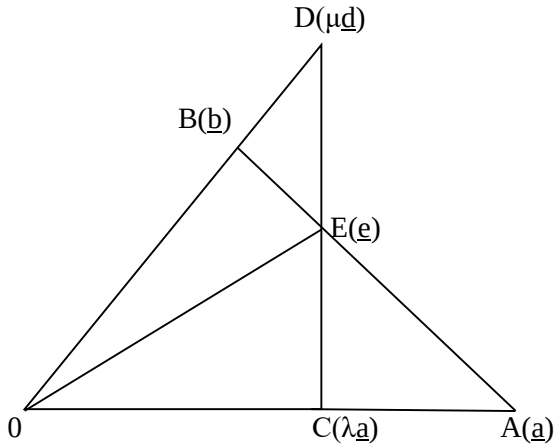
∴ B யில் நிரந்தர ஒய்விற்கு வரும் 5

$T_2 = \frac{\pi}{\omega} = \pi\sqrt{\frac{a}{2g}}$ 5

$T_1 + T_2 = \sqrt{\frac{a}{2g}}Cos^{-1}\left(\frac{1}{3}\right) + \pi\sqrt{\frac{a}{2g}}$

$= \sqrt{\frac{a}{2g}}\left[\pi + \cos^{-1}\left(\frac{1}{3}\right)\right]$ 5

(14)



$\overrightarrow{AB} = \underline{b} - \underline{a}$ 5

$\overrightarrow{AE} = \alpha_0\overrightarrow{AB}$ என்க இங்கு $0 < \alpha_0 < 1$ 5

$\overrightarrow{AE} = \alpha_0(\underline{b} - \underline{a})$ 5

$\Delta OAE \Rightarrow \overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$ 5

$= \underline{a} + \alpha_0(\underline{b} + \underline{a})$

$= (1 - \alpha_0)\underline{a} + \alpha_0\underline{b}$

$\overrightarrow{OE} = (1-\alpha)\underline{a} + \alpha b$; இங்கு $\alpha_0 = \alpha$ 5 ……(1)

$\overrightarrow{CD} = \mu\underline{b} - \lambda\underline{a}$ 5

$\overrightarrow{CE} = \beta\overrightarrow{CD}$ என்க. இங்கு $0 < \beta < 1$ 5

$\overrightarrow{CE} = \beta(\mu\underline{b} - \lambda\underline{a})$ 5

$\Delta OCE \Rightarrow \overrightarrow{OE} = \overrightarrow{OC} + \overrightarrow{CE}$

$= \lambda\underline{a} + \beta(\mu\underline{b} - \lambda\underline{a})$ 5

$\overrightarrow{OE} = \lambda(1-\beta)\underline{a} + \mu\beta\underline{b}$ 5 ……(2)

$(1)-(2) \Rightarrow (1-\alpha)\underline{a} + \alpha\underline{b} = \lambda(1-\beta)\underline{a} + \mu\beta\underline{b}$

$But \underline{a} \neq \underline{b}$

$[\underline{a}] \Rightarrow 1-\alpha = \lambda(1-\beta)$ 5

$[\underline{b}] \Rightarrow \underline{\alpha = \mu\beta}$ 5 ……(3)

கூட்ட $1 = \lambda - \lambda\beta + \mu\beta$

$1-\lambda = \beta(\mu-\lambda)$

$\beta = \dfrac{1-\lambda}{\mu-\lambda}$ 5 ……(4)

$(2) \Rightarrow \underline{e} = \lambda\left(1 - \dfrac{1-\lambda}{\mu-\lambda}\right)\underline{a} + \mu\dfrac{(1-\lambda)}{(\mu-\lambda)}\underline{b}$

$(\mu-\lambda)\underline{e} = \lambda(\mu-1)\underline{a} + (1-\lambda)\mu\underline{b}$ 5

E என்பது AB இன் நடுப்புள்ளி எனின்

$\alpha = \frac{1}{2}$ 5

$(3) \Rightarrow \beta = \dfrac{1}{2\mu}$ 5

$(4) \Rightarrow \dfrac{1}{2\mu} = \dfrac{1-\lambda}{\mu-1}$

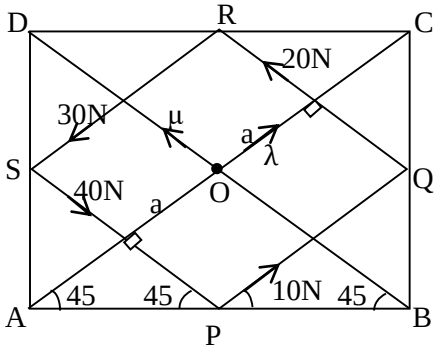
$\mu - \lambda = 2\mu - 2\lambda\mu$

$\lambda + \mu = 2\lambda\mu$

$\Rightarrow \dfrac{1}{\mu} + \dfrac{1}{\lambda} = 2$ 5

85

(b)



i. $\circlearrowleft$O G = 20 x a = 30 x a + 40 x a + 10 x a 10

G = 100a ≠ 0 5

∴ தொகுதி சமநிலையில் இருக்காது

ii. தொகுதி இணைக்கு ஒழுங்கின்

→ x=0 and ↑ y =0

→10 cos45 – 10 cos 45 – 30 cos45 + 40cos 45 + λcos45 – μcos45=0 10

λ - μ = 0

λ = μ ……(1)

↑ 20 sin45 + 20 Sin45 – 30 Sin45- 40 Sin 45 + λSin45 + μSin45 =0

$\lambda + \mu = 40$ ……..(2)

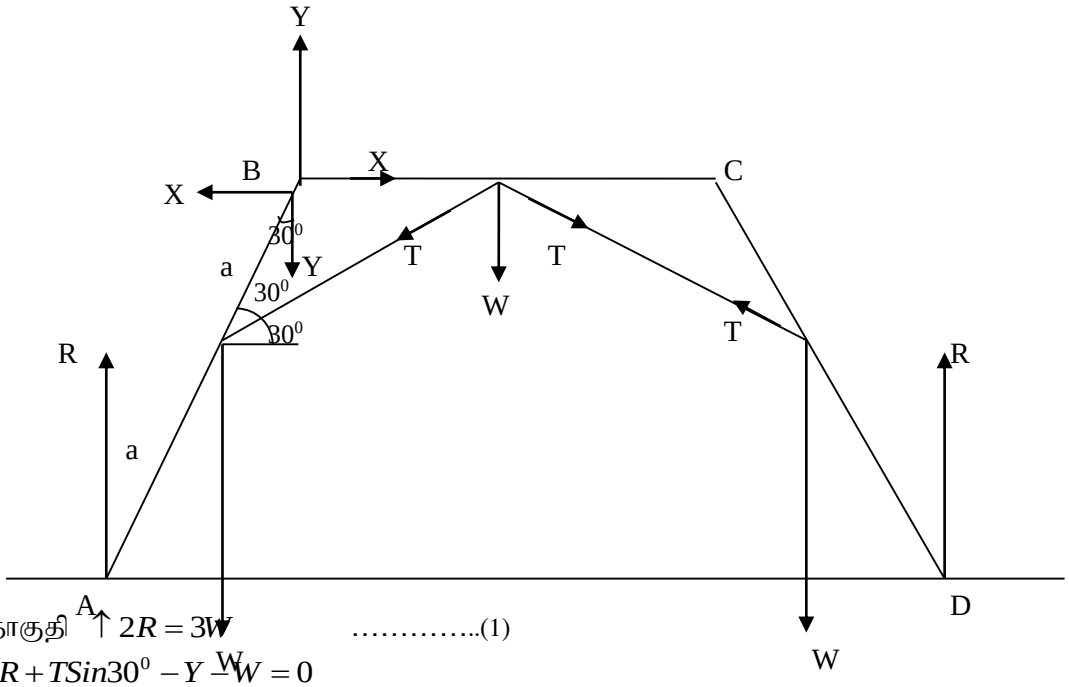
(1),(2) $\rightarrow \lambda = \mu = 20N$

iii. தொகுதி C இனூடு செல்லும் தனிவிசைக்கு ஒடுங்கின் $C\circlearrowleft = 0$

$-20 \times a + 40 \times 3a + 30 \times a + 10 \times a - \mu \times 2a = 0$

$\mu = 70N$

(15) (a)



தொகுதி $\uparrow 2R = 3W$ ……..(1)

(AB) $\uparrow R + TSin30^0 - Y - W = 0$

$\frac{1}{2}T - Y = W - \frac{3W}{2}$ கசழஅ(1)

$2Y - T = W$ ……..(2)

(AB) $\circlearrowleft T \times aSin30 + W \times aSin30^0 - R \times 2aSin30 = 0$

$T = 3W - W$

$T = 2W$ ……..(3)

$(2) \Rightarrow Y = \frac{3W}{2}$

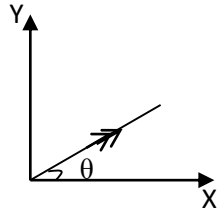
(AB) $\leftarrow X = TCos30^0$

$= \sqrt{3}W$

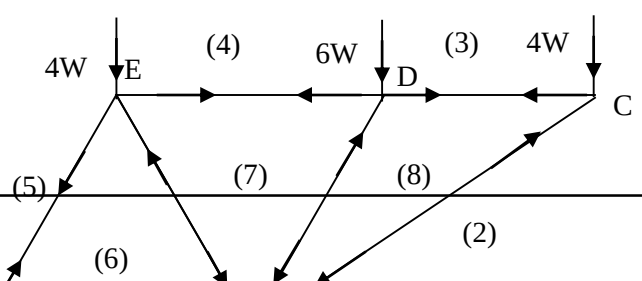
$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$



(டி)



$$R \times a = 4W \times a/2 + 6W \times \frac{3a}{2} + 4W \times \frac{5a}{2}$$

$R = 21W$

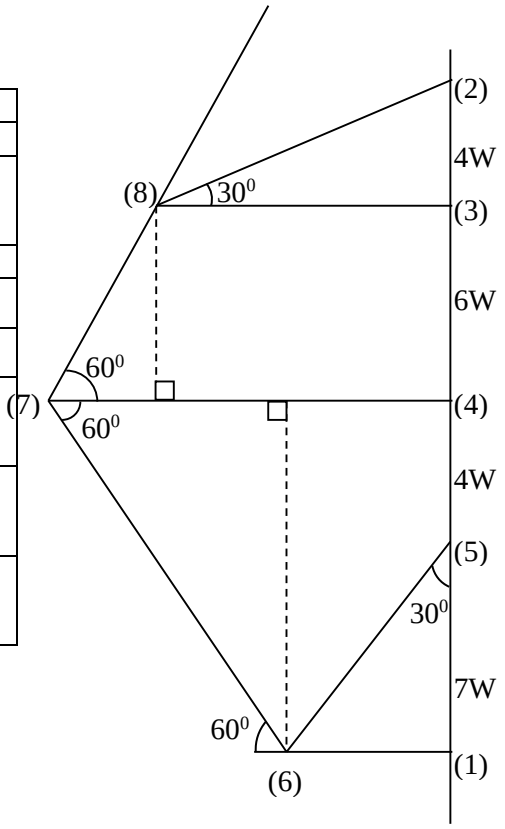
$\leftarrow X = 0$

$\uparrow R = Y + 4W + 6W + 4W$

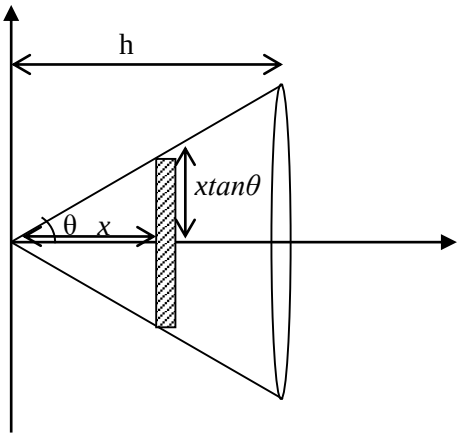
$Y = 7W$

∴ யு இல் கிடைக்கூறு ஸ்ரீ0 நிலைக்கூறு ஸ்ரீ 7று ↓

கோல்கள்	தகைப்பு	
	இழுவை	உதைப்பு
யுடீ		$\frac{7W}{\sqrt{3}}$
டீஉள		8று
உளனு	$4\sqrt{3}W$	
னுநு	$6\sqrt{3}W$	
யுநு	$\frac{14W}{\sqrt{3}}$	
டீநு		$\frac{22W}{\sqrt{3}}$
டீனு		$\frac{12W}{\sqrt{3}}$



(16)



அடர்த்தி ி α ஒ க ஸ்ரீ மஒஇ ம – மாறிலி

$\delta m = (kx)\pi(x\tan\theta)^2 \delta x$

$= k\pi \tan^2\theta x^3 \delta x$

$$\bar{x} = \frac{\sum mixi}{\sum mi}$$

$$= \frac{\int_0^h k\pi \tan^2\theta (x^3) x dx}{\int_0^h \left(k\pi \tan^2\theta\right) x^3 dx}$$

$$= \frac{\int_0^h x^4 dx}{\int_0^h x^3 dx} = \frac{x^5/5}{x^4/4}$$

$$= \frac{x^5/5}{x^4/4} = 4h/5$$

ஒ அச்சுபற்றி சமச்சீர் என்பதால் தொகுதியின் திணிவு மையம் ஒ அச்சில் இருக்கு

திணிவு ஒ அச்சிலிருந்தான திணிவு மையத்தின் தூரம்

அ1 $4h/5$

அ2 $2h$

$$(m_1 + m_2)\bar{x} = 4h/5\, m_1 + 2hm_2$$

$$\bar{x} = \left[\frac{4m_1 + 10m_2}{5(m_1 + m_2)}\right] h$$

வருளையின் வளைபரப்பு தொட்டவண்ணம் சமனிலையில் இருப்பதற்கு

$$h \le \bar{x} \le 3h$$

$$\bar{x} \le h$$

$$\frac{(4m_1 + 10m_2)h}{5(m_1 + m_2)} \ge h$$

$$4m_1 + 10m_2 \ge 5m_1 + 5m_2$$

$$m_1 \ge 5m_2$$

மிகக்குறைந்த விசையினை பிரயோகிக்கும் போது

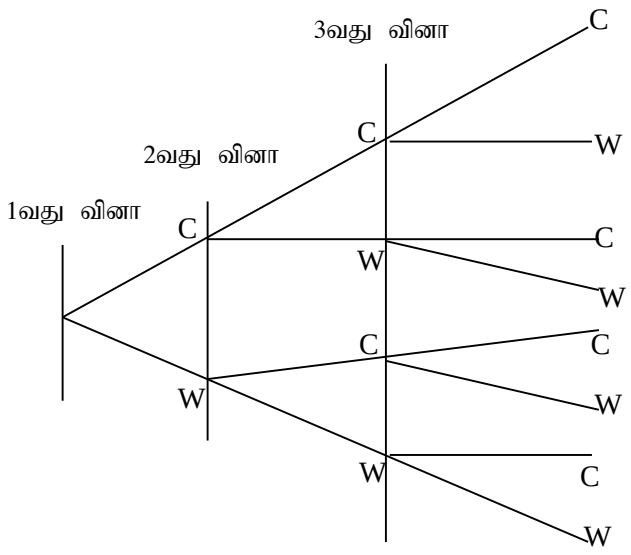
தொகுதி (யு)

$$F \times h - (m_1 + m_2) g (h - \bar{x}) = 0$$

$$F = \frac{(m_1 + m_2) g \left(h - \frac{4m_1 + 10m_2}{5(m_1 + m_2)} h\right)}{h}$$

$$= \frac{(m_1 - 5m_2) g}{5}$$

(17) (ய) ஊ : சரியாக விடையளித்தல
று : பிழையாக விடையளித்தல்



i. யு ஸ்ரீ 2வது வினாவிற்கு சரியாக விடையளித்தல்
ஸ்ரீ ஊஊஇ றுஊ
(யு) ஸ்ரீ (ஊஊ) , (றுஊ)
ஸ்ரீ 0.8 ஒ0.7 , 0.2 ஒ0.8
ஸ்ரீ 0.56 , 0.16
ஸ்ரீ 0.72

ii. (2வது வினாசரி ∴ 3வது வினாசரி) ஸ்ரீ (2வது வினாசரி ∴ 3வது வினாசரி) ∴ (3வது வினாசரி)

$$= \frac{P(CCC) + ((WCC)}{P(CCC) + P(CWC) + P(WCC) + P(WWC)}$$

$$= \frac{0.336 + 0.112}{0.336 + 0.168 + 0.112 + 0.032}$$

$$= \frac{0.448}{0.648} = \frac{448}{648}$$

$$= \frac{112}{162}$$

$$= \frac{56}{81}$$

(டி) உத்தேச இடை யு ஸ்ரீ 55 என்க.

வகுப்பாயிடை	ஒ	க	ஒ-யு ஸ்ரீ ஒ-55	$d = \frac{x - A}{C}$ ஊ ஸ்ரீ10	கன	கன^2
10-20	15	5	-40	-4	-20	80
20-30	25	12	-30	-3	-36	108
30-40	35	18	-20	-2	-36	72
40-50	45	9	-10	-1	-9	09
50-60	55	20	0	0	0	00

60-70	65	12	10	1	12	12
70-80	75	13	20	2	26	52
80-90	85	8	30	3	24	72
90-100	95	3	40	4	12	48
		$\sum F = 100$			$\sum Fd = -27$	$\sum Fd^2 = 453$

i. $\bar{x} = A + \dfrac{C\sum fd}{\sum f}$

$= 55 + \dfrac{10 \times (-27)}{100}$

$= 55 - 2.7$

$= 52.3$

ii. ஆகாரம் $= h + c\left(\dfrac{\Delta_1}{\Delta_1 + \Delta_2}\right)$

$= 50 + \dfrac{10 \times 11}{11 + 8}, \qquad \Delta_1 = 20 - 9 = 11$

$= 50 + 5.79 \qquad \Delta_2 = 20 - 12 = 8$

$= 55.79$

iii. நியம விலகல் $= c\sqrt{\dfrac{\sum fd^2}{\sum f} - \left(\dfrac{\sum fd}{\sum f}\right)^2}$

$= 10\sqrt{\dfrac{453}{100} - \left(\dfrac{-27}{100}\right)^2}$

$= 10\sqrt{453 - 0.27^2}$

$= 10\sqrt{4.457}$

$= 10 \times 2.111$

$= 21.11$

iv. ஒராய்க்குணகம். $= \dfrac{\text{இடை} - \text{ஆகாரம்}}{\text{நியமவிலகல்}}$

$= \dfrac{52.3 - 55.77}{21.11}$

ஸ்ரீ மறைப்பெறுமானம்

∴ பரம்பல் மறை ஒராயமானது

$\bar{x} = 52, \bar{y} = 48, S_x = 21, S_y = 12$

$y = ax + b$

$\bar{y} = a\bar{x} + b$

$48 = 52a + b \qquad \ldots\ldots\ldots\ldots(1)$

$S_Y = |a| S_x$

$12 = a \times 21, \therefore a > 0$

$\dfrac{4}{7} = a$

$(1) \Rightarrow 48 = 52 \times \frac{4}{7} + 6$

$6 = \frac{128}{7}$

$\therefore y = \frac{4}{7}x + \frac{128}{7}$

$x = 35 \Rightarrow y = \frac{4}{7} \times 53 + \frac{128}{7} \Rightarrow \frac{340}{7} = 48.57 < 50$

$\therefore$ பௌதிகவியலில் திறமையானவை.

இணைந்த கணிதம் I திருத்தம்

பிழை

Q1)

n = p +1 இற்கு

$$f(P+1) = 2.7^{(P+1)} + 3.5^{(P+1)}$$
$$= 2.7.7^{P} + 3.5.5^{P}$$
$$= (12+12)7^{P} + 24k + 5$$
$$= 24m + 15 \ (m \in Z)$$

Q3)

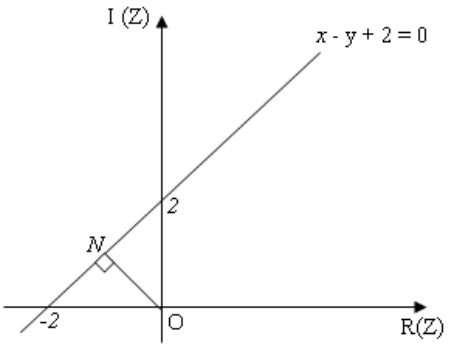
$Z = x + iy$ என்க

$$|Z + 2 + 3i| = |1 - Z|$$
$$|x + iy + 2 + 3i| = |1 - (x + iy)|$$
$$|(x+2) + (y+3)i| = |(1-x) - iy|$$
$$\Rightarrow \sqrt{(x+2)^2 + (y+3)^2} = \sqrt{(x-1)^2 + (-y)^2}$$
$$x^2 + 4x + 4 + y^2 + 6y + 9 = 1 - 2x + x^2 + y^2$$
$$6x - 6y + 12 = 0$$
$$x - y + 2 = 0$$

$$|Z|_{\min} = ON$$
$$= \frac{|0 - 0 + 2|}{\sqrt{1+1}}$$
$$= \frac{2}{\sqrt{2}}$$
$$= \sqrt{2}$$

$No \Rightarrow x + y = 0$
$x - y + 2 = 0$ $\Big\}$ $\Rightarrow$ N(-1,1)

$\therefore$ $|Z|$ இழிவாக இருக்கும். $Z = -1 + i$



Q4)

$(2x^3 - 3/x^2)^{15} = {}^{15}C_0(2x^3)^{15}(-3/x^2)^0 + {}^{15}C_1(2x^3)^{14}(-3/x^2) + {}^{15}C_2(2x^3)^{13}(-3/x^2)^2 + \ldots\ldots$

திருத்தம்

Q1)

n = p +1 இற்கு

$$f(P+1) = 2.7^{(P+1)} + 3.5^{(P+1)}$$
$$= 2.7.7^{P} + 3.5.5^{P}$$
$$= (5^{P} + 7^{P})12 + 24k + 5$$
$$= 24m + 5 \ (m \in Z)$$

Q3)

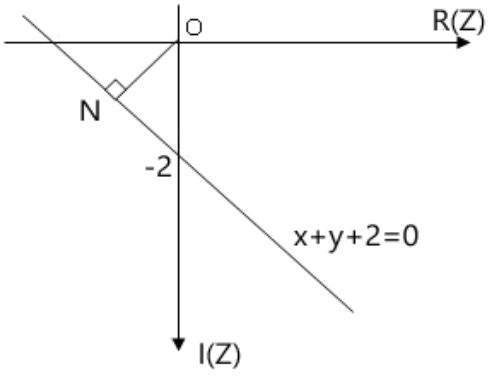
$Z = x + iy$ என்க

$$|Z + 2 + 3i| = |1 - Z|$$
$$|x + iy + 2 + 3i| = |1 - (x + iy)|$$
$$|(x+2) + (y+3)i| = |(1-x) - iy|$$
$$\Rightarrow \sqrt{(x+2)^2 + (y+3)^2} = \sqrt{(x-1)^2 + (-y)^2}$$
$$x^2 + 4x + 4 + y^2 + 6y + 9 = 1 - 2x + x^2 + y^2$$
$$6x + 6y + 12 = 0$$
$$x + y + 2 = 0$$

$$|Z|_{\min} = ON$$
$$= \frac{|0 - 0 + 2|}{\sqrt{1+1}}$$
$$= \frac{2}{\sqrt{2}}$$
$$= \sqrt{2}$$

$No \Rightarrow x - y = 0$
$x + y + 2 = 0$ $\Big\}$ $\Rightarrow$ N(-1,-1)

$\therefore$ $|Z|$ இழிவாக இருக்கும். $Z = -1 - i$



Q4) $(2x^3 - 3/x^2)^{15} = {}^{15}C_0(2x^3)^{15}(-3/x^2)^0 + {}^{15}C_1(2x^3)^{14}(-3/x^2) + {}^{15}C_2(2x^3)^{13}(-3/x^2)^2 + \ldots\ldots$

x=1 => குணகங்களின் கூட்டுத்தொகை : $(-1)^{15}$
$= (-1)$

Q5)

$$\lim_{\sin x\to 0}\frac{9}{2}\left(\frac{Sin^2(3\sin x)}{(3Sinx)^2}\right)$$

$$\frac{9}{2}(1)^2=\frac{9}{2}$$

Q5)

$$\frac{9}{2}\left\{\lim_{3\sin x\to 0}\left(\frac{Sin\ (3\sin x)}{(3Sinx)}\right)\right\}^2$$

$$\frac{9}{2}(1)^2=\frac{9}{2}$$

Q7)

$$\Rightarrow x-10y+34=0$$

Q7)

$$\Rightarrow x-9y+34=0$$

Q8)

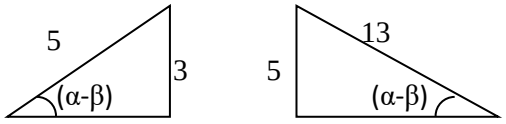
$$\pm 40=|39+\lambda|$$

Q8)

$$\pm 40=(39+\lambda)$$

Q10)

$0<(\alpha+\beta)<{}^{\pi}/_{2}$ $\qquad$ $0<\alpha+\beta<{}^{\pi}/_{2}$



$$2\alpha=(\alpha+\beta)+(\alpha-\beta)$$

$$\tan(2\alpha)=\tan(\alpha+\beta)+(\alpha-\beta)$$

$$=\frac{\tan(\alpha+\beta)+(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$=\frac{{}^{3}/_{4}+{}^{5}/_{12}}{1-{}^{3}/_{4}\times{}^{5}/_{12}}$$

$$=\left(\frac{56}{33}\right)$$

Q10)

$$\cos(\alpha+\beta)=\frac{4}{5}$$

$$\tan(\alpha+\beta)=\sqrt{\sec^2(\alpha+\beta)-1}$$

$$=\sqrt{\frac{25}{16}-1}\ =\pm\frac{3}{4}$$

$$0\le(\alpha+\beta)\le\frac{\Pi}{2}$$

எனவே $\tan(\alpha+\beta)\ge 0$

$$\tan(\alpha+\beta)=\frac{3}{4}$$

$$\sin(\alpha-\beta)=\frac{5}{13}$$

$$\tan(\alpha-\beta)=\frac{\sin(\alpha-\beta)}{\sqrt{1-\sin^2(\alpha-\beta)}}=$$

$$=\frac{{}^{5}/_{13}}{\sqrt{1-{}^{25}/_{169}}}=\pm\frac{5}{12}$$

$$0\le(\alpha-\beta)\le\frac{\Pi}{4}$$

எனவே $\tan(\alpha-\beta)\ge 0$

$$\tan(\alpha-\beta)=\frac{5}{12}$$

$$2\alpha=(\alpha+\beta)+(\alpha-\beta)$$

$$\tan(2\alpha)=\tan(\alpha+\beta)+(\alpha-\beta)$$

$$=\frac{\tan(\alpha+\beta)+(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$=\frac{{}^{3}/_{4}+{}^{5}/_{12}}{1-{}^{3}/_{4}\times{}^{5}/_{12}}$$

$$=\left(\frac{56}{33}\right)$$

Part B

பிழை	திருத்தம்
Q_{11} (a) (ii) $9x^4 - 3(\alpha+\beta)x^3 + (18+\alpha\beta)x$ $+9 \equiv 9(x^4+x^2+1)$ (b) a – b + -1 -10 = 48 a = b $f(x) = (x-1)(x-2)(2x+1)(3x+5)$	$9x^4 - 3(\alpha+\beta)x^3 + (18+\alpha\beta)x^2 - 3(\alpha+\beta)x + 9 = 9(x^4+x^2+1)$ $a - b + c - 1 - 10 = 48$ a = 6 $f(x) = (x-1)(x-2)(2x+1)(3x-5)$
Q_{12} (a) (i) 3rd line $= 5_{C_1} \times (6_{C_1} \times 2_{C_1}) \times 7_{C1} \times 3_{C_1}$ =180 மொத்தம் =3180 (b) *let* $Ur = r^3 - 3r + 1$ $\therefore U_{r+1} = (r+1)^3 - 3(r+1) + 1$ $U_r + U_{r+1} = r^3 - 3r + 1 + (r+1)^3 - 3(r+1) + 1$ $= r^3 - 3r + 1 + r^3 + 3r^2 + 3r + 1 - 3r - 3 + 1$ $= 2r^3 + 3r^2 - 3r$ $= \frac{n^4 + 2n^3 + n^2 - 3n^2 - 3n + 2n}{2}$	$= {}^5C_1 \times {}^6C_1 \times {}^2C_1 \times {}^7C_1 \times {}^3C_1 = 1260$ மொத்தம் =3260 $U_r + U_{r+1} = r(2r^2+3r-3)$ ----------(*) $U_r = Ar^3 + Br^2 + Cr + D$ என்க (*)-----$Ar^3 + Br^2 + Cr + D + [A(r+1)^3 + B(r+1)^2 + C(r+1) + D]$ $= r(2r^2+3r-3)$ $Ar^3 + Br^2 + Cr + D + [A(r^3+3r^2+3r+1) + B(r^2+2r+1) + c(r+1) + D = 2r^3 + 3r^2 - 3r$ r^3இன் குணகம் :- 2A = 2 => A = 1 r^2இன் குணகம் :- 3A+2B= 3 => B = 0 r இன் குணகம் :- 3A+2B+2C= -3 => C= -3 மாறிலி :- A+B+C+2D= -3 => D= 1 $U_r = Ar^3 + Br^2 + Cr + D$ $= r^3 - 3r + 1$ $= \frac{n^4 + 2n^3 - 5n^2 - 4n}{4}$

Q13 (a) (ii)

$$AB = \begin{pmatrix} -2+2 & 2+3 \\ 0+6 & 0+3 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 5 \\ 6 & 3 \end{pmatrix}$$

$$(C^{-1}C)\times\begin{pmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ 6 & 3 \end{pmatrix}$$

$$IX = \begin{pmatrix} 0-9 & -\frac{5}{2}-\frac{9}{2} \\ 0+12 & 5+6 \end{pmatrix}$$

$$X = \begin{pmatrix} -9 & -7 \\ 12 & 11 \end{pmatrix}$$

$$AB = \begin{pmatrix} -2+2 & -2+1 \\ 0+6 & 0+3 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & -1 \\ 6 & 3 \end{pmatrix}$$

$$(C^{-1}C)X = \begin{pmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 6 & 3 \end{pmatrix}$$

$$IX = \begin{pmatrix} (0-9) & (\frac{1}{2}-\frac{9}{2}) \\ (0+12) & (-1+6) \end{pmatrix}$$

$$X = \begin{pmatrix} -9 & -4 \\ 12 & 5 \end{pmatrix}$$

Q14) (a)

$$\frac{d(\tan^{-1}x)}{dy} = \frac{1}{(1+x^2)}\frac{dx}{dy}$$

↓ (Cannot be written directly)

$$\frac{dy}{d(\tan^{-1}x)} = (1+x^2)\frac{dx}{dy}$$

$$\frac{d(\tan^{-1}x)}{dy} = \frac{1}{(1+x^2)}\frac{dx}{dy}$$

$$\frac{dy}{d(\tan^{-1}x)} = \frac{1}{\frac{d(\tan^{-1}x)}{dx}}$$

$$= \frac{1}{\frac{1}{1+x^2}(\frac{dx}{dy})}$$

$$= (1+x^2)\frac{dy}{dx}$$

(b) $f'(x) = \dfrac{(2x+1)(2x+1)+2(x^2+x)}{x^2(x+1)^2}$

$$= \frac{-2(x^2+x+\frac{1}{2})}{x^2(x+1)^2} = \frac{-2(x+\frac{1}{2})^2+\frac{1}{4}}{x^2(x+1)^2}$$

$$f'(x) = \frac{x(x+1)2-(1+2x)(2x+1)}{x^2(x+1)^2}$$

$$= \frac{-2[(x+\frac{1}{2})^2+\frac{1}{4}]}{x^2(x+1)^2}$$

Q15) (a)

$= \frac{3}{2}\left(\ln(x^2+1) + \tan^{-1}(x)\right) - \frac{3}{2}\ln(x-1) + C$

$= \frac{3}{4}\ln(x^2+1) + \frac{3}{2}\tan^{-1}(x) - \frac{3}{2}\ln|x-1| - \frac{1}{(x-1)} + c$

(b)

$2I = \int_a^{\Pi} \frac{\Pi(1+\sin x)}{1+\sin x}\,dx$

$2I = \int_a^{\Pi} \frac{\pi(1+\sin x) - \pi}{1+\sin x}\,dx$

$2I = \Pi^2 - \frac{\Pi}{2}\tan(\frac{\Pi}{4} - \frac{x}{2})\Big|_0^{\Pi}$

$2I = \Pi^2 - \frac{\Pi}{2}\frac{\tan(\frac{\Pi}{4} - \frac{x}{2})}{-\frac{1}{2}}\Big|_0^{\Pi}$

$2I = \Pi^2 - \frac{\Pi}{2}[\tan(\frac{\Pi}{4} - \frac{\Pi}{2}) - \tan(\frac{\Pi}{4})$

$2I = \Pi^2 + \frac{2\Pi}{2}[\tan(\frac{\Pi}{4} - \frac{\Pi}{2}) - \tan(\frac{\Pi}{4})$

$2I = \Pi^2 - \frac{\Pi}{2}(-1-1)$

$2I = \Pi^2 - \frac{2\Pi}{2}(-1-1)$

(c)

$A = \int_{-1}^{1} \frac{x^2}{1+e^x}\,dx$

$A = \int_{-1}^{1} \frac{x^2}{1+e^{-x}}\,dx$

(d)

$= \frac{1}{2}\ln(\frac{3}{2}) - \frac{1}{2}\ln\frac{3}{2} + \frac{3}{4}$

$= \frac{1}{2}\ln(\frac{3}{2}) - \frac{1}{2}(4\ln\frac{3}{2} - \frac{3}{2})$

$= -\frac{3}{2}\ln(\frac{3}{2}) + \frac{3}{4}$

$= \frac{3}{2}\ln(\frac{2}{3}) + \frac{3}{4}$

Q16)

$N \equiv \left\{\frac{x_0+\alpha}{2}\right\} + b\left\{\frac{y_0+\beta}{2}\right\} + C = 0$

$N \equiv \left\{\frac{x_0+\alpha}{2}\right\}, b\left\{\frac{y_0+\beta}{2}\right\} = 0$

$I \equiv \left\{\alpha - \frac{2a(a\alpha+b\beta+c}{a^2+b^2}\,\beta - \frac{2b(a\alpha+b\beta+c}{a^2+b^2}\right\}$

$I \equiv \left\{\alpha - \frac{2a(a\alpha+b\beta+c}{a^2+b^2}, \beta - \frac{2b(a\alpha+b\beta+c}{a^2+b^2}\right\}$

$\underline{3y = 3}$

$\underline{3y = x}$

$B \equiv \left\{-\frac{7t}{5}, t - \frac{t}{5}\right\}$

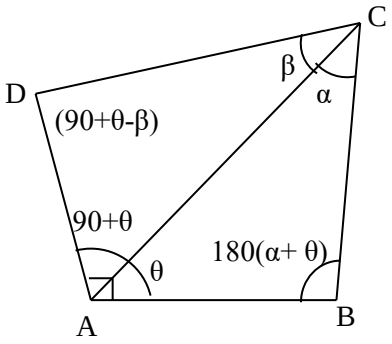
$B \equiv \left\{-\frac{7t}{5}, -\frac{t}{5}\right\}$

Q_{17}) (a)

$$= \frac{(Cos2x - Sin2x) + \frac{1}{\sqrt{2}}(\cos x - Sinx)}{(Cos2x + Sin2x + 2Sinx.\cos x) + \frac{1}{\sqrt{2}}(Sinx + Cosx)}$$

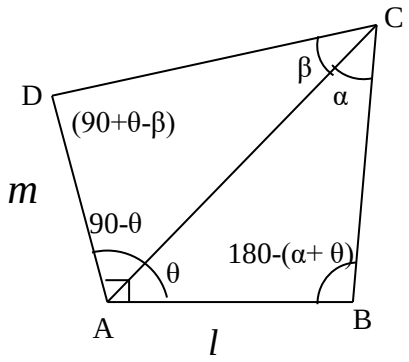
$$= \frac{(Cos^2x - Sin^2x) + \frac{1}{\sqrt{2}}(\cos x - Sinx)}{(Cos^2x + Sin^2x + 2Sinx.\cos x) + \frac{1}{\sqrt{2}}(Sinx + Cosx)}$$

(b)



$$\frac{l}{SinA} = \frac{AC}{Sin[180 - (\alpha + \theta)]} \Rightarrow AC = \frac{Sin(\alpha + \theta)}{Sin\alpha} l$$

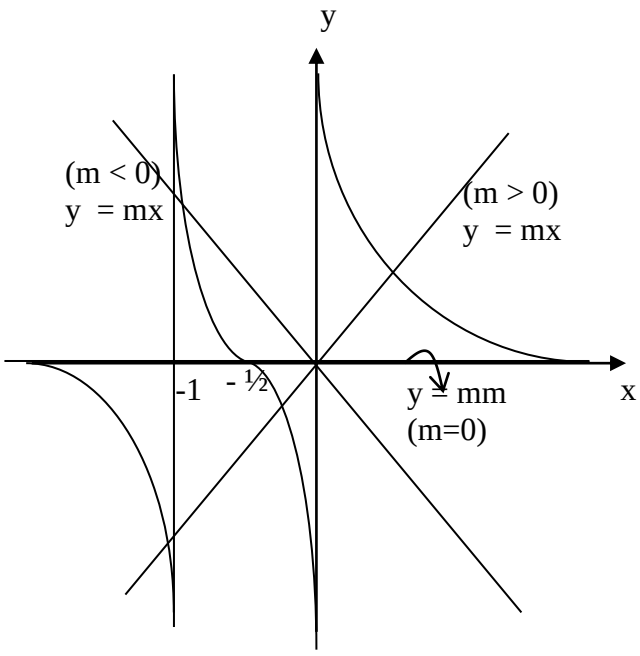
$$\frac{A}{Sin\beta} = \frac{AC}{Sin[90 - (\theta - \beta)]} \Rightarrow AC = \frac{Cos(\theta - \beta)}{Sin\beta} m$$

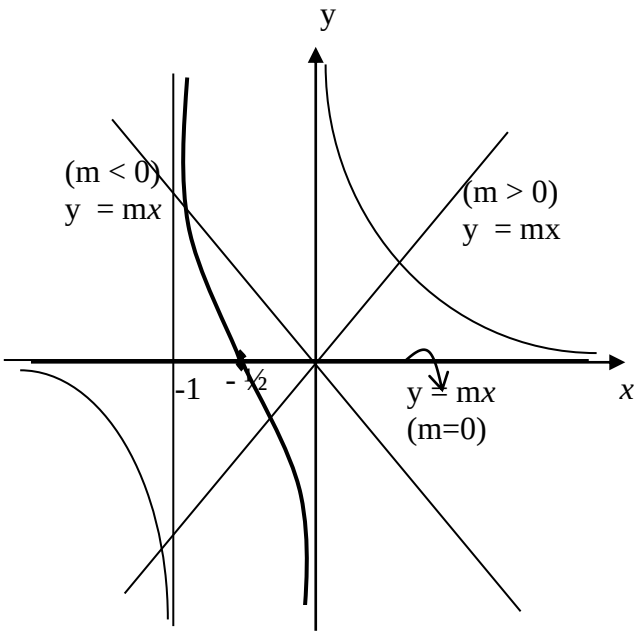


$$\frac{l}{Sin\alpha} = \frac{AC}{Sin[180 - (\alpha + \theta)]} \Rightarrow AC = \frac{Sin(\alpha + \theta)}{Sin\alpha} l$$

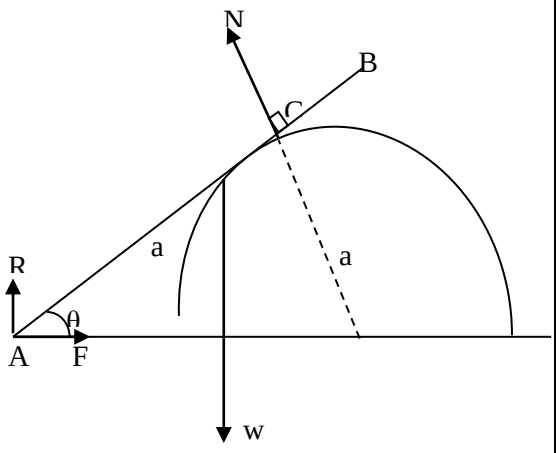
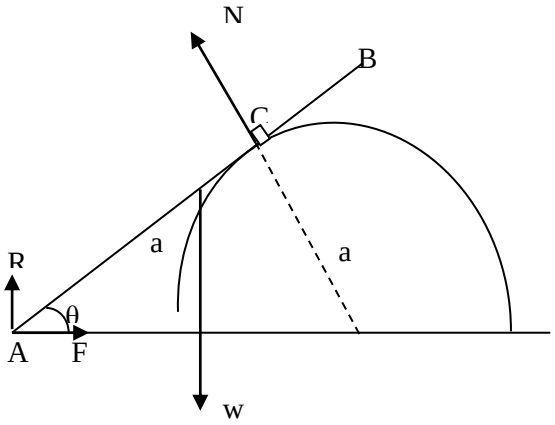
$$\frac{m}{Sin\beta} = \frac{AC}{Sin[90 - (\theta - \beta)]} \Rightarrow AC = \frac{Cos(\theta - \beta)}{Sin\beta} m$$

Q_{14})(b)





இணைந்த கணிதம் II திருத்தம்

Q03) $(1)=(2)\Rightarrow ku^3+MgV\sin\alpha$	 $(1)=(2)\Rightarrow ku^3=MgV\sin\alpha+kV^3$
Q05) $\overrightarrow{CD}=(2\mu-\lambda)-(2\lambda+\mu)\underline{j}$ $\left[\overrightarrow{CD}\right]=\sqrt{(2\mu-d)^2+(2\lambda+\mu)^2}=2\sqrt{10}$ $(2\mu-d)^2+(2\lambda+\mu)^2=40$ $5(\mu^2+\lambda^2)=10$	$\overrightarrow{CD}=(2\mu-\lambda)\underline{i}-(2\lambda+\mu)\underline{j}$ $\left[\overrightarrow{CD}\right]=\sqrt{(2\mu-\lambda)^2+(2\lambda+\mu)^2}=2\sqrt{10}$ $(2\mu-\lambda)^2+(2\lambda+\mu)^2=40$ $5(\mu^2+\lambda^2)=40$
Q06)  	
Q09) $\bar{x}=33.33$	$\bar{x}=36.67$
Q10) $d=10$ எனின் $e=1,\ f=12$ ஆகும் இது (1)' இற்கு அமைவாகும்	$d=10$ எனின் $e=11,\ f=12$ ஆகும் இது (1)' இற்கு அமைவாகும்
Q13) $V^2=\omega^2(a^2-x^2)$ $x=a/4$ இன் $V=\sqrt{ag}$ $\cos=\frac{a/4}{3a/4}=\frac{1}{3}$ $\ddot{y}=-2g/a\ y-a/4=2$ $x=a/4$ அலைவு மையம்	$V^2=\omega^2(a^2-X^2)$ $x=0 \Rightarrow X=a/4$ இன் $V=\sqrt{ag}$ $\cos\theta=\frac{a/4}{3a/4}=\frac{1}{3}$ $BD=\frac{3a}{4}-\frac{a}{4}=\frac{a}{2}$ (இவ்வரி அச்சிடப்படவில்லை) $\ddot{y}=-2g/a\ (y-a/4)$ $\ddot{y}=0 \Rightarrow y=a/4$ அலைவு மையம்
Q14) (b) ↑ 20 sin45 + **20** Sin45 – 30 Sin45- 40 Sin 45 + λSin45 + μSin45 =0	 ↑ 20 sin45 + 10 Sin45 – 30 Sin45- 40 Sin 45 + λSin45 + μSin45 =0

$Q_{17)}$ (a) ஊ : சரியாக விடையளித்தல று : பிழையாக விடையளித்தல	C : சரியாக விடையளித்தல W : பிழையாக விடையளித்தல
(b) $x = 35 \Rightarrow y = \frac{4}{7} \times 53 + \frac{128}{7} \Rightarrow \frac{340}{7} = 48.57 < 50$	$x = 53 \Rightarrow y = \frac{4}{7} \times 53 + \frac{128}{7} \Rightarrow \frac{340}{7} = 48.57 < 50$